2.3 Kombinační čísla a jejich vlastnosti

① Vyjádřete jediným kombinačním číslem:

a) $\binom{6}{2}+\binom{6}{3}=$ použijeme vzorec $\binom{n}{k}+\binom{n}{k+1}=\binom{n+1}{k+1}$

$=\underline{\binom{7}{3}}$

b) $\binom{7}{2}+\binom{7}{3}=\underline{\binom{8}{3}}$ c) $\binom{12}{5}+\binom{12}{6}=\underline{\binom{13}{6}}$

d) $\binom{25}{5}+\binom{25}{4}=\binom{25}{4}+\binom{25}{5}=\underline{\binom{26}{5}}$

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② Vyjádřete jediným kombinačním číslem:

a) $\binom{11}{4}+\binom{11}{6}$ → Použijeme vzorec pro počet doplňkových kombinací: $\binom{n}{k}=\binom{n}{n-k}$

$=\binom{11}{4}+\binom{11}{11-6}=\binom{11}{4}+\binom{11}{5}=\underline{\underline{\binom{12}{5}}}$

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b) $\binom{14}{3}+\binom{14}{10}=\binom{14}{3}+\binom{14}{14-10}=\binom{14}{3}+\binom{14}{4}=\underline{\underline{\binom{15}{4}}}$

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c) $\binom{17}{8}+\binom{17}{8}=\binom{17}{8}+\binom{17}{17-8}=\binom{17}{8}+\binom{17}{9}=\underline{\underline{\binom{18}{9}}}$

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d) $\binom{8}{6}+\binom{8}{3}=\binom{8}{8-6}+\binom{8}{3}=\binom{8}{2}+\binom{8}{3}=\underline{\underline{\binom{9}{3}}}$

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③ $\binom{3}{3}+\binom{4}{3}+\binom{5}{3}+\binom{6}{3}+\binom{7}{3}=\binom{8}{4}$ Dokažte tuto rovnost.

a) ↳ nahraďme $\binom{3}{3}$ komb. číslem $\binom{4}{4}$, neboť $\binom{3}{3}=\binom{4}{4}=1$

$\binom{4}{4}+\binom{4}{3}=\binom{4}{3}+\binom{4}{4}=\binom{5}{4}$ … řešíme postupně dílčími součty

$\binom{5}{4}+\binom{5}{3}=\binom{6}{4}$ … $\binom{6}{4}+\binom{6}{3}=\binom{7}{4}$; $\binom{7}{3}+\binom{7}{4}=\binom{8}{4}$

$L=\binom{8}{4}$, $P=\binom{8}{4}$, $L=P \Rightarrow$ Rovnost platí.

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b) $\binom{8}{8}+\binom{9}{8}+\binom{10}{8}+\binom{11}{8}+\binom{12}{8}=\binom{13}{9}$

$\downarrow$

nahradíme ekvivalentem $\binom{9}{9}$

$$\binom{9}{9}+\binom{9}{8}=\binom{10}{9}$$

$$\binom{10}{8}+\binom{10}{9}=\binom{11}{9}\dots\binom{11}{8}+\binom{11}{9}=\binom{12}{9}\dots\binom{12}{8}+\binom{12}{9}=\binom{13}{9}$$

$L=\binom{13}{9}$, $P=\binom{13}{9}$; $L=P \Rightarrow$ Rovnost platí.

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4) Řešte rovnici:

a) $\binom{x+1}{2}+\binom{x}{2}=4$

$$\frac{(x+1)\cdot x}{2\cdot 1}+\frac{x\cdot(x-1)}{2\cdot 1}=4 \quad |\cdot 2$$

$$x(x+1)+x\cdot(x-1)=8$$

$$x^2+x+x^2-x=8$$

$$2x^2=8$$

$$x^2=4$$

$$x=\pm 2$$

-2 nevyhovuje $\Rightarrow$ $\boxed{x=2}$

Zkouška:

$L=\binom{2+1}{2}+\binom{2}{2}=\binom{3}{2}+\binom{2}{2}=3+1=4$

$P=4$, $L=P$

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b) $\binom{x-1}{x-2}+\binom{x-2}{x-4}=1$ Použijeme vzorec pro počet doplň. kombinací.

$$\binom{x-1}{x-1-(x-2)}+\binom{x-2}{x-2-(x-4)}=1$$

$$\binom{x-1}{1}+\binom{x-2}{2}=1$$

$$x-1+\frac{(x-2)\cdot(x-3)}{2}=1 \quad |\cdot 2$$

$$2x-2+x^2-5x+6=2$$

$$x^2-3x+2=0$$

$$x_{1,2}=\frac{3\pm\sqrt{9-8}}{2}=\frac{3\pm 1}{2}=\begin{cases}2\\1\end{cases}$$

Provedeme-li zkoušku, zjistíme, že vypočítané hodnoty x nevyhovují dané rovnici.

$L=\binom{2-1}{2-2}+\binom{2-2}{2-4}=\binom{1}{0}+\binom{0}{-4}$ atd.

$\downarrow$

nelze

Rovnice nemá řešení.

2) ____

c) $\binom{x}{2}+\binom{x-1}{x-3}=16$

$\binom{x}{2}+\binom{x-1}{x-1-(x-3)}=16$

$\binom{x}{2}+\binom{x-1}{2}=16$

$\frac{x\cdot(x-1)}{2}+\frac{(x-1)\cdot(x-2)}{2}=16 \quad |\cdot 2$

$x(x-1)+(x-1)\cdot(x-2)=32$

$x^2-x+x^2-3x+2=32$

$2x^2-4x-30=0 \quad |:2$

$x^2-2x-15=0$

$x_{1,2}=\frac{2\pm\sqrt{64}}{2}=\frac{2\pm 8}{2}=\begin{cases} 5 \\ -3\ (\text{ne}) \end{cases}$

Zk.

$L=\binom{5}{2}+\binom{5-1}{5-3}=\binom{5}{2}+\binom{4}{2}=10+6=16$

$P=16$; $L=P$

Rovnice má řešení $\boxed{x=5}$.

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d) $\binom{x-2}{x-4}+\binom{x-3}{x-5}=16$

$\binom{x-2}{x-2-(x-4)}+\binom{x-3}{x-3-(x-5)}=16$

$\binom{x-2}{2}+\binom{x-3}{2}=16$

$\frac{(x-2)\cdot(x-3)}{2}+\frac{(x-3)\cdot(x-4)}{2}=16 \quad |\cdot 2$

$(x-2)\cdot(x-3)+(x-3)\cdot(x-4)=32$

$2x^2-12x-14=0 \quad |:2$

$x^2-6x-7=0$

$x_{1,2}=\frac{6\pm\sqrt{64}}{2}=\frac{6\pm 8}{2}=\begin{cases} 7 \\ -1\ (\text{ne}) \end{cases}$

Zkouška:

$L=\binom{5}{3}+\binom{4}{2}=10+6=16$, $P=16$

$L=P$ Řešení: $\boxed{x=7}$

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5) Řešte rovnici:

a) $\binom{x+2}{x}\cdot\binom{x}{0}-\binom{x-1}{x-3}\cdot\binom{x}{x}=\frac{1}{2}x^2-8$

$\binom{x+2}{x+2-x}\cdot 1-\binom{x-1}{x-1-(x-3)}\cdot 1=\frac{1}{2}x^2-8$

$\binom{x+2}{2}-\binom{x-1}{2}=\frac{1}{2}x^2-8$

$\frac{(x+2)\cdot(x+1)}{2}-\frac{(x-1)\cdot(x-2)}{2}=\frac{1}{2}x^2-8 \quad |\cdot 2$

$(x+2)\cdot(x+1)-(x-1)\cdot(x-2)=x^2-16$

$x^2+3x+2-(x^2-3x+2)=x^2-16$

$6x=x^2-16$

$x^2-6x-16=0$

$x_{1,2}=\frac{6\pm\sqrt{100}}{2}=\frac{6\pm 10}{2}=\begin{cases} 8 \\ (-2)\ \text{ne} \end{cases}$

Řešení: $\boxed{x=8}$

Zk.

$L=\binom{10}{8}\cdot\binom{8}{0}-\binom{7}{5}\cdot\binom{8}{8}=$

$45\cdot 1-21\cdot 1=45-21=24$

$P=\frac{1}{2}\cdot 64-8=24$

$L=P$

b) $\binom{x}{x-2}+\binom{x}{x-1}=\binom{6}{4}$

$\binom{x}{x-(x-2)}+\binom{x}{x-(x-1)}=15$

$\binom{x}{2}+\binom{x}{1}=15$

$\frac{x(x-1)}{2}+x=15 \quad |\cdot 2$

$x^2-x+2x-30=0$

$x^2+x-30=0$

$x_{1,2}=\frac{-1\pm\sqrt{121}}{2}=\frac{-1\pm 11}{2}=\begin{cases}5\\ -6\ (\text{ne})\end{cases}$

Zkouška:

$L=\binom{5}{3}+\binom{5}{4}=\binom{6}{4}$, $P=\binom{6}{4}$; $L=P$

Řešení: $\boxed{x=5}$

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c) $4\cdot\binom{x+1}{x-1}+15=10\cdot\binom{x+1}{x}-3$

$4\cdot\binom{x+1}{x+1-(x-1)}+15=10\binom{x+1}{x+1-x}-3$

$4\cdot\binom{x+1}{2}+15=10\binom{x+1}{1}-3$

$4\cdot\frac{(x+1)\cdot x}{2}+15=10\cdot(x+1)-3$

$2x(x+1)+15=10x+10-3$

$2x^2+2x+15=10x+7$

$2x^2-8x+8=0 \quad |:2$

$x^2-4x+4=0$

$x_{1,2}=\frac{4\pm 0}{2}$

$\boxed{x=2}$ Zkouška vynechána.

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d) $\binom{x-1}{x-3}+\binom{x-2}{x-4}=9$

$\binom{x-1}{x-1-(x-3)}+\binom{x-2}{x-2-(x-4)}=9$

$\binom{x-1}{2}+\binom{x-2}{2}=9$

$\frac{(x-1)\cdot(x-2)}{2}+\frac{(x-2)\cdot(x-3)}{2}=9 \quad |\cdot 2$

$(x-1)\cdot(x-2)+(x-2)\cdot(x-3)=18$

$x^2-3x+2+x^2-5x+6=18$

$2x^2-8x-10=0 \quad |:2$

$x^2-4x-5=0$

$x_{1,2}=\frac{4\pm\sqrt{36}}{2}=\frac{4\pm 6}{2}=\begin{cases}5\\ -1\ (\text{ne})\end{cases}$

Zkouška:

$L=\binom{4}{2}+\binom{3}{1}=6+3=9$

$P=9$, $L=P$

Řešení: $\boxed{x=5}$

⑥ Určete všechna přirozená n, pro která platí:

a) $2\cdot\binom{n+1}{n-1}+n(n-5)=6$

$2\cdot\binom{n+1}{n+1-(n-1)}+n^2-5n=6$

$2\cdot\binom{n+1}{2}+n^2-5n=6$

$2\cdot\dfrac{(n+1)\cdot n}{2}+n^2-5n=6$

$n^2+n+n^2-5n=6$

$2n^2-4n-6=0 \quad |:2$

$n^2-2n-3=0$

$n_{1,2}=\dfrac{2\pm\sqrt{16}}{2}=\dfrac{2\pm4}{2}=\begin{cases}3\\-1\ (\text{ne})\end{cases}$

Zkouška:

$L=2\cdot\binom{4}{2}+3(3-5)=12-6=6$

$P=6\ ,\ L=P$

Platí pro $\boxed{n=3}$.

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b) $\binom{n-1}{2}+\binom{n-2}{2}=9$

$\dfrac{(n-1)\cdot(n-2)}{2}+\dfrac{(n-2)\cdot(n-3)}{2}=9 \quad |\cdot 2$

$(n-1)\cdot(n-2)+(n-2)\cdot(n-3)=18$

$n^2-3n+2+n^2-5n+6=18$

$2n^2-8n-10=0 \quad |:2$

$n^2-4n-5=0$

$n_{1,2}=\dfrac{4\pm\sqrt{36}}{2}=\dfrac{4\pm6}{2}=\begin{cases}5\\-1\ (\text{ne})\end{cases}$

Zkouška:

$L=\binom{4}{2}+\binom{3}{2}=6+3=9$

$P=9\ ,\ L=P$

Platí pro $\boxed{n=5}$.

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c) $2\cdot\binom{n+4}{n+2}-4n=16$

$2\cdot\binom{n+4}{n+4-(n+2)}-4n=16$

$2\cdot\binom{n+4}{2}-4n=16$

$2\cdot\dfrac{(n+4)\cdot(n+3)}{2}-4n=16$

$(n+4)\cdot(n+3)-4n=16$

$n^2+7n+12-4n-16=0$

$n^2+3n-4=0$

$n_{1,2}=\dfrac{-3\pm\sqrt{25}}{2}=\dfrac{-3\pm5}{2}=\begin{cases}1\\-4\ (\text{ne})\end{cases}$

Zk.

$L=2\cdot\binom{5}{3}-4\cdot1=2\cdot10-4=16\ ,\ P=16$

$L=P$

Platí pro $\boxed{n=1}$.

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d) $\binom{n-1}{n-3} - n = 8$

$\binom{n-1}{n-1-(n-3)} - n = 8$

$\binom{n-1}{2} - n = 8$

$\frac{(n-1)\cdot(n-2)}{2} - n = 8 \quad |\cdot 2$

$(n-1)\cdot(n-2) - 2n = 16$

$n^2 - 3n + 2 - 2n - 16 = 0$

$n^2 - 5n - 14 = 0$

$n_{1,2} = \frac{5 \pm \sqrt{81}}{2} = \frac{5 \pm 9}{2} = \begin{cases} 7 \\ -2 \text{ (ne)} \end{cases}$

Zkouška:

$L = \binom{6}{4} - 7 = 15 - 7 = 8, \ P = 8, \ L = P$

Platí pro $n = 7$.

KONEC ČLÁNKU 2.3