1.5 Logaritmické a exponenciální rovnice (příklady ze sbírky pro OA - str. 67-69)

V příkladech 1 až 4 určete podmínky řešitelnosti a pak řešte logaritmické rovnice.

1 (a) $\log(x+1) - \log(x-1) = \log 4$ ... $x > 1$ ... $(x+1>0) \wedge (x-1>0)$

$\log \frac{x+1}{x-1} = \log 4$

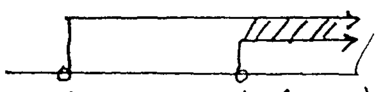
$\frac{x+1}{x-1} = 4 \quad | \cdot (x-1)$

$x+1 = 4x-4$

$3x = 5$

$\boxed{x = \frac{5}{3}\ ;\ x > 1}$

$x > -1 \wedge x > 1$



$-1 \quad 1 \quad (1; +\infty)$

(b) $\log(3x-1) - \log(2x+2) = -\log 2$

$\log(3x-1) = \log(2x+2) - \log 2$

$\log(3x-1) = \log \frac{2x+2}{2}$

$3x-1 = \frac{2x+2}{2}$

$3x-1 = x+1$

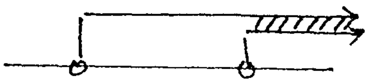
$2x = 2$

$\boxed{x = 1\ ;\ x > \frac{1}{3}}$

$3x-1>0 \wedge 2x+2>0$

$3x>1 \wedge 2x>-2$

$x > \frac{1}{3}$



-2

(c) $\log(4x-2) - \log 3 = 1$

$\log(4x-2) - \log 3 = \log 10$

$\log \frac{4x-2}{3} = \log 10$

$\frac{4x-2}{3} = 10$

$4x-2 = 30$

$4x = 32$

$\boxed{x = 8\ ;\ x > \frac{1}{2}}$

$4x-2>0$

$4x>2$

$x > \frac{1}{2}$

(d) $\log(x+2) + \log(x-7) = 2 \cdot \log(x-4)$

$\log(x+2)\cdot(x-7) = \log(x-4)^2$

$(x+2)\cdot(x-7) = x^2 - 8x + 16$

$x^2 + 2x - 7x - 14 = x^2 - 8x + 16$

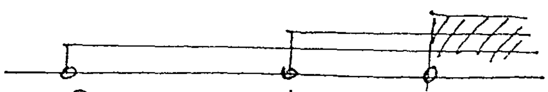
$3x = 30$

$x = 10$

$\boxed{x = 10\ ;\ x > 7}$

$x+2>0 \wedge x-7>0 \wedge x-4>0$

$x>-2 \wedge x>7 \wedge x>4$



$-2 \quad 4 \quad 7$

$x > 7$

ⓔ $\log(x+4)+\log(x-2)=2\cdot\log(x+3)$

$x+4>0 \wedge x-2>0 \wedge x+3>0$

$\underbrace{x>-4 \wedge x>2 \quad \wedge x>-3}$

$\Downarrow$

Podmínka $\boxed{x>2}$

$\log(x+4)\cdot(x-2) = \log(x+3)^2$

$\log(x^2-2x-8) = \log(x^2+6x+9)$

$\cancel{x^2}+2x-8 = \cancel{x^2}+6x+9$

$-4x = 17$

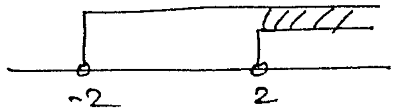
$x=-\frac{17}{4}$

Protože logaritmy záporných čísel neexistují, tak úloha nemá řešení.

ⓕ $\log(x+2)+\log(x-2)=2\cdot\log(x-2)$

$x+2>0 \wedge x-2>0$

$x>-2 \wedge x>2$



$x>2$

$\log(x+2)+\log(x-2) = \log(x-2)^2$

$\log(x^2-4) = \log(x-2)^2$

$x^2-4 = x^2-4x+4$

$4x=8$

$x=2$, a to je v rozporu s podmínkou $x>2$.

Úloha nemá řešení!

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② ⓐ $2\cdot\log(x-2)=\log(14-x)$

$x-2>0 \wedge 14-x>0$

$-x>-14$

$\underbrace{x>-2 \wedge x<14}$

$\boxed{x\in(-2;14)} \dots -2<x<14$

$(x-2)^2 = 14-x$

$x^2-4x+4 = 14-x$

$x^2-3x-10 = 0$

$x_{1,2}=\frac{3\pm\sqrt{49}}{2}=\frac{3\pm7}{2}=$ $\boxed{5}$; -2 ; $-2\notin(-2;14) \Rightarrow$ není řešením rovnice.

ⓑ $\log(x+13)-\log(x-3)=1-\log 2$

$x+13>0 \wedge x-3>0$

$\underbrace{x>-13 \wedge x>3}$

$x>3$

$\log\frac{x+13}{x-3}+\log 2 = 1$

$\log\left(\frac{x+13}{x-3}\cdot 2\right) = \log 10$

$\frac{(x+13)\cdot 2}{x-3} = 10$

$2x+26 = 10x-30$

$-8x = -56$

$\boxed{x=7 \ ; \ x>3}$

ⓒ $\log(x+2) - \log(x-1) = 2 - \log 4$

$x+2>0 \wedge x-1>0$

$x>-2 \wedge x>1 \Rightarrow x>1$

$\log\frac{x+2}{x-1} + \log 4 = 2$

$\log\left(\frac{x+2}{x-1}\cdot 4\right) = \log 100$

$\frac{4x+8}{x-1} = 100$

$4x+8 = 100x - 100$

$-96x = -108$

$\boxed{x = \frac{9}{8} ;\ x>1}$

ⓓ $\log(7x+6) = 1 + \log(3x-4)$

$7x+6>0 \wedge 3x-4>0$

$7x>-6 \wedge 3x>4$

$x>-\frac{6}{7} \wedge x>\frac{4}{3}$

$\boxed{x>\frac{4}{3}}$

$\log(7x+6) - \log(3x-4) = 1$

$\log\frac{7x+6}{3x-4} = \log 10$

$\frac{7x+6}{3x-4} = 10$

$7x+6 = 30x-40$

$-23x = -46$

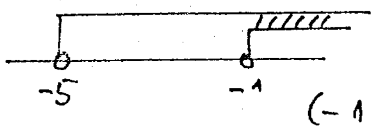
$\boxed{x = 2}$

ⓔ $\frac{\log(2x+10)}{2} = \log(x+1)$

$2x+10>0 \wedge x+1>0$

$2x>-10$

$x>-5 \wedge x>-1$



$(-1;\infty)$

$\log(2x+10) = 2\cdot\log(x+1)$

$\log(2x+10) = \log(x+1)^2$

$2x+10 = (x+1)^2$

$\cancel{2x}+10 = x^2+\cancel{2x}+1$

$x^2 = 9$

$x_{1,2} = \pm 3$ $\quad(-3 \notin (-1;\infty))$ není řešením

$\boxed{x=3 ;\ x>-1 \text{ nebo } x\in(-1;+\infty)}$

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③ ⓐ $\frac{\log 2}{\log(4x-15)} = 2$

$4x-15>0$

$4x>15$

$x>\frac{15}{4}$

$x>3{,}75$

$\log 2 = 2\cdot\log(4x-15)$

$\log 2 = \log(4x-15)^2$

$2 = (4x-15)^2$

$2 = 16x^2-120x+225$

$16x^2-120x+223 = 0$

$\boxed{x \doteq 4{,}1 ;\ x>3{,}75}$

$x_{1,2} = \frac{120\pm\sqrt{128}}{32} \doteq$ 4,103553391 ; 3,396446609 nevyhovuje, neboť $x>3{,}75$

ⓑ $\frac{3+\log x}{2-\log x} = 4$ $\quad x>0 \quad 2-\log x \neq 0$

$-\log x \neq -2$

$\log x \neq 2 \Rightarrow x \neq 100$

$3+\log x = 4(2-\log x)$

$3+\log x = 8-4.\log x$

$\log x = 5 - \log x^4$

$\log x + \log x^4 = 5$

$\log x^5 = \log 100\,000$

$x^5 = 100\,000$

$x^5 = 10^5$

$\boxed{x = 10 \ ; \ x>0, \ x \neq 100}$

ⓒ $\frac{1+\log x}{\log x} - \frac{1-\log x}{\log x} = 1$

$\frac{1+\log x - (1-\log x)}{\log x} = 1$

$\frac{1+\log x - 1 + \log x}{\log x} = 1$

$\frac{2.\log x}{\log x} = 1$

$2 = 1$

Rovnice nemá řešení.

ⓓ $\log(x-2) + \log(x+2) = 2.\log(2-x)$

$\log(x^2-4) = \log(2-x)^2$

$x^2-4 = (2-x)^2$

$\cancel{x^2}-4 = 4-4x+\cancel{x^2}$

$4x = 8$

$x = 2$, Tento kořen je však v rozporu s podmínkou:

$x-2>0 \ \wedge \ x+2>0 \ \wedge \ 2-x>0$

$x>2 \ \wedge \ x>-2 \ \wedge \ x<2$

Úloha nemá řešení.

④ ⓐ $\log x + \frac{4}{\log x} = 4 \quad |.\log x \ ; \ x>0$

$\log^2 x + 4 = 4.\log x$

$\log^2 x - 4\log x + 4 = 0$

* Substituce: $\log x = y$

$y^2 - 4y + 4 = 0$

$y_{1,2} = \frac{4 \pm \sqrt{16-16}}{2} = 2$

$\boxed{y=2}$ zpět do *

$\log x = 2$

$\log x = \log 100$

$x = 100$

$\boxed{x = 10^2 \ ; \ x>0}$

Zkouška:

$L = \log 100 + \frac{4}{\log 100} = 2 + \frac{4}{2} = 4$

$P = 4; \ L = P$

(b) $x^{\log x} = 10\,000$ Rovnici zlogaritmujeme.

$\log x^{\log x} = \log 10\,000$

$\log x \cdot \log x = 4$

$\log^2 x = 4$

Substituce: $\log x = y$

$y^2 = 4$

$y_1 = 2, y_2 = -2$

Zpět:

$\log x_1 = 2 \qquad \log x_2 = -2$

$x_1 = 100 \qquad x_2 = \frac{1}{100}$

Podm. $x > 0$

$\boxed{x_1 = 100,\ x_2 = \frac{1}{100}\ (0{,}01),\ x > 0}$

(c) $x^{3+4\log x} = 10x^6$ zlogaritmujeme

$\log x^{3+4\log x} = \log 10x^6$

$(3+4\cdot\log x)\cdot\log x = \log 10 + \log x^6$

$3\cdot\log x + 4\cdot\log^2 x = 1 + 6\cdot\log x$

$4\log^2 x - 3\log x - 1 = 0$

Substituce: $\log x = y$

$4y^2 - 3y - 1 = 0$

$y_{1,2} = \frac{3 \pm \sqrt{25}}{8} = \frac{3 \pm 5}{8} = \begin{cases} y_1 = 1 \\ y_2 = -\frac{1}{4} \end{cases}$

$\log x_1 = 1 \qquad \log x_2 = -\frac{1}{4}$

$x_1 = 10^1 \qquad \boxed{x_2 = 10^{-\frac{1}{4}}}$

$\boxed{x_1 = 10} \qquad \boxed{x > 0}$

(d) $x^{3-\log x} = 100$

$\log x^{3-\log x} = \log 100$

$(3-\log x)\cdot\log x = 2$

$\log^2 x - 3\log x + 2 = 0$

Sub.: $\log x = y$

$y^2 - 3y + 2 = 0$

$y_{1,2} = \frac{3 \pm \sqrt{9-8}}{2} = \frac{3 \pm 1}{2} = \begin{cases} y_1 = 2 \\ y_2 = 1 \end{cases}$

$\log x_1 = 2 \qquad \log x_2 = 1$

$x_1 = 10^2 \qquad x_2 = 10^1$

$\boxed{x_1 = 100} \ \boxed{x_2 = 10} \ \boxed{x > 0}$

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(5) V oboru reálných čísel řešte danou rovnici. Zapište podmínky platnosti pro x.

(a) $\log x + \frac{16}{\log x} = -6 \quad | \cdot \log x\,; \ x > 0$

$\log^2 x + 16 = -6\log x$

$\log^2 x + 6\log x + 16 = 0$

substituce: $\log x = t$

$t^2 + 6t + 16 = 0$

$t_{1,2} = \frac{-6 \pm \sqrt{36-64}}{2} = \frac{-6 \pm \sqrt{-28}}{2}$

diskrim. $D < 0 \Rightarrow$ v R

nemá rovnice řešení.

(b) $\log x + \dfrac{3}{\log x} = 4 \quad |\cdot \log x \quad ; x>0$

$\log^2 x + 3 = 4\log x$

$\log^2 x - 4\log x + 3 = 0$

sub. $\log x = t$

$t^2 - 4t + 3 = 0$

$t_{1,2} = \dfrac{4 \pm \sqrt{16-12}}{2} = \dfrac{4 \pm 2}{2} = \begin{cases} t_1 = 3 \\ t_2 = 1 \end{cases}$

$\log x_1 = 3 \qquad \log x_2 = 1$

$x_1 = 10^3 \qquad x_2 = 10^1$

$x_1 = 1000$	$x_2 = 10$	$x>0$

Zkouška:

$x_1 = 1000$

$L = \log 1000 + \dfrac{3}{\log 1000} = 3+1 = 4$

$P = 4 \;, L = P$

$x_2 = 10$

$L = \log 10 + \dfrac{3}{\log 10} = 1+3 = 4$

$P = 4 \;, L = P$

(c) $\dfrac{\log(x^2+12)}{\log(6-x)} = 2 \quad |\ldots$ Podmínka $6-x>0$

$-x > -6$

$x < 6$

$\log(x^2+12) = 2 \cdot \log(6-x)$

$\log(x^2+12) = \log(6-x)^2$

$x^2 + 12 = (6-x)^2$

$x^2 + 12 = 36 - 12x + x^2$

$12x = 24$

$\boxed{x = 2 \;; x<6}$

Zkouška: $L = \dfrac{\log(2^2+12)}{\log(6-2)} = \dfrac{\log 16}{\log 4} = 2$

na kalkulačce, nebo

$P = 2 \;; L = P$ pomocí vzorce na

str. 13: $\log_4 16 = 2$

(d) $\dfrac{\log(x^2+3)}{\log(x+3)} = 2 \quad |\cdot \log(x+3) \quad \ldots \quad x+3>0$

$x > -3$

$\log(x^2+3) = 2\log(x+3)$

$\log(x^2+3) = \log(x+3)^2$

$x^2 + 3 = (x+3)^2$

$x^2 + 3 = x^2 + 6x + 9$

$6x = -6$

$\boxed{x = -1 \;; x>-3}$

Zkouška:

$L = \dfrac{\log[(-1)^2+3]}{\log(-1+3)} = \dfrac{\log 4}{\log 2} = 2$ (na kalk.)

nebo pomocí vzorce na str. 13

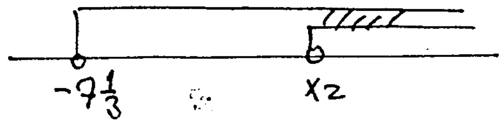
$\log_2 4 = 2$

e) $\frac{1}{2}\log(3x+22) = \log(x-2)$

$(3x+22>0) \wedge (x-2>0)$

$3x > -22 \wedge x>2$

$x > -\frac{22}{3} \wedge x>2$



$\log(3x+22)^{\frac{1}{2}} = \log(x-2)$

$\sqrt{3x+22} = x-2$

$\left(\sqrt{3x+22}\right)^2 = (x-2)^2$

$3x+22 = x^2-4x+4$

$x^2-7x-18=0$

$x_{1,2} = \dfrac{7\pm\sqrt{121}}{2} = \dfrac{7\pm 11}{2} = \begin{cases} x_1 = 9 \\ \underbrace{x_2=-2}_{\text{nevyhovuje podmínce}} \end{cases} \Big\} x>2$

Zkouška: $L = \frac{1}{2}\log(27+22) = \frac{1}{2}\log 49 = \log 49^{\frac{1}{2}} = \log\sqrt{49} = \log 7$

$P = \log(9-2) = \log 7 \qquad L=P$

Řešení: $\boxed{x=9\ ;\ x>2}$

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f) $\frac{1}{2}\log(2x+7) = \log(x-4)$

$2x+7>0 \wedge x-4>0$

$2x>-7 \wedge x>4$

$x>-3{,}5 \wedge x>4 \Rightarrow x>4$

$\log(2x+7)^{\frac{1}{2}} = \log(x-4)$

$\sqrt{2x+7} = (x-4) \quad)^2$

$2x+7 = (x-4)^2$

$2x+7 = x^2-8x+16$

$x^2-10x+9=0$

$x_{1,2} = \dfrac{10\pm\sqrt{64}}{2} = \dfrac{10\pm 8}{2} = \begin{cases} x_1 = 9 \\ \underbrace{x_2 = 1}_{\text{nevyhovuje podmínce}} \end{cases}$

Zk. $L = \frac{1}{2}\log(18+7) = \frac{1}{2}\log 25 = \log 25^{\frac{1}{2}} = \log\sqrt{25} = \log 5$

$P = \log(9-4) = \log 5 \qquad L=P$

$\boxed{x_1 = 9;\ x>4}$

Exponenciální rovnice

V příkladech 6 až 10 řešte exponenciální rovnice:

6 a) $3^x = 27$

$3^x = 3^3$

$\boxed{x = 3}$

b) $2^{x+1} = 16$

$2^x \cdot 2 = 2^4 \quad | :2$

$2^x = 2^3$

$\boxed{x = 3}$

c) $3^{2x+2} = 81$

$3^{2x} \cdot 3^2 = 81 \quad | :9$

$3^{2x} = 9$

$(3^2)^x = 9$

$9^x = 9^1$

$\boxed{x = 1}$

d) $10^{7x+2} = 100$

$10^{7x} \cdot 10^2 = 100 \quad | :100$

$(10^7)^x = 1$

$(10^7)^x = 10^0$

$\boxed{x = 0}$

e) $5^{\frac{x}{2}} = 125$

$5^{\frac{x}{2}} = 5^3$

$\frac{x}{2} = 3$

$\boxed{x = 6}$

f) $4^{x-1} = 16$

$4^x \cdot 4^{-1} = 16$

$4^x \cdot \frac{1}{4} = 16$

$4^x = 64$

$4^x = 4^3$

$\boxed{x = 3}$

g) $2^x = \frac{1}{4}$

$2^x = 4^{-1}$

$2^x = (2^2)^{-1}$

$2^x = 2^{-2}$

$\boxed{x = -2}$

h) $3^{x-1} = \frac{1}{27}$

$3^x \cdot 3^{-1} = \frac{1}{3^3}$

$3^x \cdot 3^{-1} = 3^{-3}$

$3^x \cdot \frac{1}{3} = \frac{1}{3^3}$

$3^x = \frac{3}{3^3}$

$3^x = \frac{1}{3^2}$

$3^x = 3^{-2}$

$\boxed{x = -2}$

ch) $4^{2x+7} = \frac{1}{256}$

$4^{2x} \cdot 4^7 = \frac{1}{4^4} \quad | \cdot \frac{1}{4^7}$

$4^{2x} = \frac{1}{4^{11}}$

$4^{2x} = 4^{-11}$

$2x = -11$

$\boxed{x = -\frac{11}{2}}$

7 a) $\left(\frac{2}{3}\right)^x = \frac{8}{27}$

$\left(\frac{2}{3}\right)^x = \frac{2^3}{3^3}$

$\left(\frac{2}{3}\right)^x = \left(\frac{2}{3}\right)^3$

$\boxed{x = 3}$

b) $\left(\frac{2}{3}\right)^x = \frac{27}{8}$

$\left(\frac{2}{3}\right)^x = \left(\frac{3}{2}\right)^3$

$\left(\frac{2}{3}\right)^x = \left(\frac{2}{3}\right)^{-3}$

$\boxed{x = -3}$

c) $\left(\frac{3}{4}\right)^{x-1} = \frac{16}{9}$

$\left(\frac{3}{4}\right)^{x-1} = \left(\frac{4}{3}\right)^2$

$\left(\frac{3}{4}\right)^{x-1} = \left(\frac{3}{4}\right)^{-2}$

$x - 1 = -2$

$\boxed{x = -1}$

d) $3 \cdot 4^x = 48 \quad | :3$

$4^x = 16$

$4^x = 4^2$

$\boxed{x = 2}$

e) $5 \cdot 2^{x+2} = 320 \quad | :5$

$2^{x+2} = 64$

$2^{x+2} = 2^6$

$x + 2 = 6$

$\boxed{x = 4}$

f) $3 \cdot 7^{x+5} = 7203 \quad | :3$

$7^{x+5} = 2401$

$7^{x+5} = 7^4$

$x + 5 = 4$

$\boxed{x = -1}$

8 a)

$2^{x^2-6x-2,5}=16\sqrt{2}$

$2^{x^2-6x-2,5}=2^4\cdot 2^{\frac{1}{2}}$

$2^{x^2-6x-2,5}=2^{\frac{9}{2}}$

$x^2-6x-2,5-4,5=0$

$x^2-6x-7=0$

$x_{1,2}=\frac{6\pm\sqrt{64}}{2}=\begin{cases}7\\-1\end{cases}$

$\boxed{x_1=7,\ x_2=-1}$

b) $3^3\cdot 27^{2x-3}=81^{3x-5}$

$3^3\cdot(3^3)^{2x-3}=(3^4)^{3x-5}$

$3^3\cdot 3^{6x-9}=3^{(12x-20)}$

$3^{(3+6x-9)}=3^{(12x-20)}$

$3^{6x-6}=3^{12x-20}$

$6x-6=12x-20$

$6x=14$

$\boxed{x=\frac{7}{3}}$

c) $4^{\sqrt{x+1}}=64\cdot 2^{\sqrt{x+1}}$

$(2^2)^{\sqrt{x+1}}=2^6\cdot 2^{\sqrt{x+1}}$

$2^{2\sqrt{x+1}}=2^{6+\sqrt{x+1}}$

$2\sqrt{x+1}=6+\sqrt{x+1}$

$2\sqrt{x+1}-\sqrt{x+1}=6$

$\sqrt{x+1}=6$

$x+1=36$

$\boxed{x=35}$

d) $9^{x^2+5x+3}=3^{x^2+4x-2}$

$(3^2)^{x^2+5x+3}=3^{x^2+4x-2}$

$3^{2x^2+10x+6}=3^{x^2+4x-2}$

$2x^2+10x+6=x^2+4x-2$

$x^2+6x+8=0$

$x_{1,2}=\frac{-6\pm\sqrt{4}}{2}=\begin{cases}-4\\-2\end{cases}$

$\boxed{x_1=-4}\ \boxed{x_2=-2}$

9 a) $\frac{2^{x+3}}{6^{7-x}}\cdot\frac{3^{x+2}}{8^{x-1}}=\frac{9^{x-2}}{3}$

$2^{x+3}\cdot 6^{x-7}\cdot 3^{x+2}\cdot 8^{1-x}=9^{x-2}\cdot 3^{-1}$

$2^{x+3}\cdot(2\cdot 3)^{x-7}\cdot 3^{x+2}\cdot(2^3)^{1-x}=(3^2)^{x-2}\cdot 3^{-1}$

$2^{x+3}\cdot 2^{x-7}\cdot 3^{x-7}\cdot 3^{x+2}\cdot 2^{3-3x}=3^{2x-4}\cdot 3^{-1}$

$2^{x+3+x-7+3-3x}\cdot 3^{x-7+x+2}=3^{2x-4-1}$

$2^{-x-1}\cdot 3^{2x-5}=3^{2x-5}\quad |:3^{2x-5}$

$2^{-x-1}=1$

$2^{-x-1}=2^0$

$-x-1=0$

$\boxed{x=-1}$

b) $5^{x+1}+5^{x+2}=30$

$5^x\cdot 5+5^x\cdot 5^2=30\quad |:5$

$5^x+5^x\cdot 5=6$

$5^x(1+5)=6$

$6\cdot 5^x=6\quad |:6$

$5^x=1$

$5^x=5^0$

$\boxed{x=0}$

c) $3^{x+4}-3^{x+3}+3^{x+2}-3^{x+1}+3^x=4941$

$3^x\cdot 3^4-3^x\cdot 3^3+3^x\cdot 3^2-3^x\cdot 3+3^x=4941$

$3^x\cdot(3^4-3^3+3^2-3+1)=4941$

$3^x\cdot 61=4941\quad |:61$

$3^x=81$

$3^x=3^4\ \Rightarrow\ x=4$

ⓐ $5\cdot 4^{x+1}-240=4^{x+2}+4^{x-1}$

$5\cdot 4^x\cdot 4-240=4^x\cdot 16+4^x\cdot \frac{1}{4} \quad |\cdot 4$

$80\cdot 4^x-960=64\cdot 4^x+4^x$

$80\cdot 4^x-960=65\cdot 4^x$

$15\cdot 4^x=960 \quad |:15$

$4^x=64$

$4^x=4^3$

$\boxed{x=3}$

ⓔ $3^{2x}-12\cdot 3^x+27=0$

$(3^x)^2-12\cdot 3^x+27=0$

Substituce: $3^x=y$

$y^2-12y+27=0$

$y_{1,2}=\frac{12\pm\sqrt{36}}{2}=\frac{12\pm 6}{2}=\begin{cases} y_1=9 \\ y_2=3\end{cases}$

$3^x=9 \qquad 3^x=3$

$3^x=3^2 \qquad 3^x=3^1$

$\boxed{x_1=2} \qquad \boxed{x_2=1}$

ⓕ $2^{2x}-12\cdot 2^x+32=0$

$(2^x)^2-12\cdot 2^x+32=0$

Sub. $2^x=y$

$y^2-12y+32=0$

$y_{1,2}=\frac{12\pm\sqrt{16}}{2}=\begin{cases} 8 \\ 4\end{cases}$

$2^x=8 \qquad 2^x=4$

$2^x=2^3 \qquad 2^x=2^2$

$\boxed{x_1=3} \qquad \boxed{x_2=2}$

ⓖ $3^{x-1}=5$

$3^x\cdot 3^{-1}=5$

$3^x\cdot \frac{1}{3}=5$

$3^x=15$

Protože 15 nelze vyjádřit jako mocninu 3 s celým exponentem, tak musíme vzniklou rovnici zlogaritmovat.

$\log 3^x=\log 15$

$x\cdot\log 3=\log 15$

$x=\frac{\log 15}{\log 3} \quad \boxed{x\doteq 2{,}464973521}$

ⓗ $4^{x-2}=5\cdot 3^{x-2}$

$4^x\cdot\frac{1}{16}=5\cdot 3^x\cdot\frac{1}{9}$

$\frac{4^x}{16}=\frac{5\cdot 3^x}{9}$

$9\cdot 4^x=80\cdot 3^x$

Zlogaritmujeme

$\log 9\cdot 4^x=\log 80\cdot 3^x$

$\log 9+\log 4^x=\log 80+\log 3^x$

$\log 9+x\cdot\log 4=\log 80+x\cdot\log 3$

$x\cdot\log 4-x\cdot\log 3=\log 80-\log 9$

$x\cdot(\log 4-\log 3)=\log 80-\log 9$

$x=\frac{\log 80-\log 9}{\log 4-\log 3}$ (na kalkulačce)

$\boxed{x\doteq 7{,}59450194}$

10 a

$$3\cdot 4^x + \frac{9^{x+2}}{3} = 6\cdot 4^{x+1} - \frac{9^{x+1}}{2} \quad |\cdot 6$$

$$18\cdot 4^x + 2\cdot(9^{x+2}) = 36(4^{x+1}) - 3\cdot(9^{x+1})$$

$$18\cdot 4^x + 2\cdot 9^x\cdot 81 = 36\cdot 4^x\cdot 4 - 3\cdot 9^x\cdot 9$$

$$18\cdot 4^x + 162\cdot 9^x = 144\cdot 4^x - 27\cdot 9^x$$

$$162\cdot 9^x + 27\cdot 9^x = 144\cdot 4^x - 18\cdot 4^x$$

$$189\cdot 9^x = 126\cdot 4^x \quad |:9$$

$$21\cdot 9^x = 14\cdot 4^x \quad |:7$$

$$3\cdot 9^x = 2\cdot 4^x$$

$$\frac{9^x}{4^x} = \frac{2}{3}$$

$$\left(\frac{9}{4}\right)^x = \frac{2}{3}$$

$$\log\left(\frac{9}{4}\right)^x = \log\frac{2}{3}$$

$$x\cdot\log\frac{9}{4} = \log\frac{2}{3}$$

$$x = \frac{\log\frac{2}{3}}{\log\frac{9}{4}} \quad \text{(na kalk.)}$$

$$\boxed{x = -\frac{1}{2}}$$

b

$$0{,}25^{2-x} = \frac{256}{2^{x+3}}$$

$$\left(\frac{1}{4}\right)^{2-x} = \frac{256}{2^{x+3}} \quad |\cdot 2^{x+3}$$

$$\frac{1}{16}\cdot\left(\frac{1}{4}\right)^{-x}\cdot 2^{x+3} = 256$$

$$\frac{1}{16}\cdot 4^x\cdot 2^x\cdot 8 = 256$$

$$\frac{4^x\cdot 2^x}{2} = 256$$

$$8^x = 512$$

$$8^x = 8^3$$

$$\boxed{x = 3}$$

c

$$\sqrt[x-1]{\sqrt[3]{2^{3x-1}}} - \sqrt[3x-7]{8^{x-3}} = 0$$

$$\sqrt[3(x-1)]{2^{3x-1}} - \sqrt[3x-7]{8^{x-3}}$$

$$2^{\frac{3x-1}{3x-3}} - 8^{\frac{x-3}{3x-7}} = 0$$

$$2^{\frac{3x-1}{3x-3}} = (2^3)^{\frac{x-3}{3x-7}}$$

$$2^{\frac{3x-1}{3x-3}} = 2^{\frac{3x-9}{3x-7}}$$

$$\frac{3x-1}{3x-3} = \frac{3x-9}{3x-7}$$

$$(3x-1)\cdot(3x-7) = (3x-9)\cdot(3x-3)$$

$$9x^2 - 3x - 21x + 7 = 9x^2 - 27x - 9x + 27$$

$$12x = 20$$

$$x = \frac{20}{12}$$

$$\boxed{x = \frac{5}{3}}$$

d

$$\sqrt[2x+3]{4^{3-x}} = 1024$$

$$4^{\frac{3-x}{2x+3}} = 4^5$$

$$\frac{3-x}{2x+3} = 5$$

$$3 - x = 10x + 15$$

$$11x = -12$$

$$\boxed{x = -\frac{12}{11}}$$

ⓔ $2^{5x-3} = 3^{3x-2}$

$2^{5x} \cdot \frac{1}{8} = 3^{3x} \cdot \frac{1}{9} \quad | \cdot 72$

$2^{5x} \cdot 9 = 3^{3x} \cdot 8$

$(2^5)^x \cdot 9 = (3^3)^x \cdot 8$

$32^x \cdot 9 = 27^x \cdot 8$

$\frac{32^x}{27^x} = \frac{8}{9}$

$\left(\frac{32}{27}\right)^x = \frac{8}{9}$

$\log\left(\frac{32}{27}\right)^x = \log\frac{8}{9}$

$x \cdot \log\frac{32}{27} = \log\frac{8}{9}$

$x = \log\frac{8}{9} : \log\frac{32}{27}$

$\boxed{x \doteq -0{,}693\,253\,11}$

ⓕ $\left(\frac{1}{3}\right)^{2-3x} = 5^x$

$3^{3x-2} = 5^x$

$(3^3)^x \cdot \frac{1}{9} = 5^x$

$27^x = 9 \cdot 5^x$

$\left(\frac{27}{5}\right)^x = 9$

$\log\left(\frac{27}{5}\right)^x = \log 9$

$x \cdot \log\frac{27}{5} = \log 9$

$x = \log 9 : \log\frac{27}{5}$

$\boxed{x \doteq 1{,}302\,909\,12}$

ⓖ $3^x + 3^{x+1} + 3^{x+2} = 5^x + 5^{x+1} + 5^{x+2}$

$3^x + 3^x \cdot 3 + 3^x \cdot 9 = 5^x + 5^x \cdot 5 + 5^x \cdot 25$

$3^x(1+3+9) = 5^x(1+5+25)$

$13 \cdot 3^x = 31 \cdot 5^x$

$\left(\frac{3}{5}\right)^x = \frac{31}{13}$

$\log\left(\frac{3}{5}\right)^x = \log\frac{31}{13}$

$x \cdot \log\frac{3}{5} = \log\frac{31}{13}$

$x = \log\frac{31}{13} : \log\frac{3}{5}$

$\boxed{x = -1{,}701\,241\,689}$

---

⑪ Určete souřadnice průsečíku grafů funkcí $f(x), g(x)$, je-li:

ⓐ $f(x) = 5 \cdot 2^{x+2} - 6 \cdot 3^{x+2}$

$g(x) = 3^{x+3} + 2 \cdot 2^{x+1}$

$\underbrace{f(x)} = 5 \cdot 2^x \cdot 4 - 6 \cdot 3^x \cdot 9$

$y_1 = 20 \cdot 2^x - 54 \cdot 3^x$

$\underbrace{g(x)} = 3^x \cdot 27 + 2 \cdot 2^x \cdot 2$

$y_2 = 27 \cdot 3^x + 4 \cdot 2^x$

$y_1 = y_2$

$20 \cdot 2^x - 54 \cdot 3^x = 27 \cdot 3^x + 4 \cdot 2^x$

$16 \cdot 2^x = 81 \cdot 3^x$

$\left(\frac{2}{3}\right)^x = \frac{81}{16}$

$\log\left(\frac{2}{3}\right)^x = \log\frac{81}{16}$

$x \cdot \log\frac{2}{3} = \log\frac{81}{16}$

$x = \log\frac{81}{16} : \log\frac{2}{3}$

$\boxed{x = -4}$ dále str. 13

$f(x)=y_1=5\cdot 2^{-4+2}-6\cdot 3^{-4+2}=5\cdot 2^{-2}-6\cdot 3^{-2}=5\cdot\frac{1}{4}-6\cdot\frac{1}{9}=\frac{5}{4}-\frac{6}{9}=\boxed{\frac{7}{12}}$

Ověření pro $g(x)$

$g(x)=y_2=3^{-4+3}+2\cdot 2^{-4+1}=3^{-1}+2\cdot 2^{-3}=\frac{1}{3}+2\cdot\frac{1}{8}=\frac{1}{3}+\frac{1}{4}=\boxed{\frac{7}{12}}$

Grafy obou funkcí se protínají v bodě

$\boxed{[-4;\frac{7}{12}]}$

(b) $f(x)=4^{x-1}-10$

$g(x)=2\cdot(4^{x-2}+11)$

$\underbrace{f(x)}_{y_1}=4^x\cdot\frac{1}{4}-10$ $\qquad$ $\underbrace{g(x)}_{y_2}=2\cdot(4^x\cdot\frac{1}{16}+11)$

$y_1=\frac{1}{4}\cdot 4^x-10$ $\qquad$ $y_2=\frac{1}{8}\cdot 4^x+22$

$y_1=y_2$

$\frac{1}{4}\cdot 4^x-10=\frac{1}{8}\cdot 4^x+22$

$\frac{1}{4}\cdot 4^x-\frac{1}{8}\cdot 4^x=32$

$\frac{1}{8}4^x=32$

$4^x=256$

$4^x=4^4$

$\boxed{x=4}$

$f(x)=y_1=4^4\cdot\frac{1}{4}-10=4^3-10=\boxed{54}$

Ověření pro $g(x)$:

$g(x)=2\cdot(4^{4-2}+11)=2(4^2+11)=2\cdot 27=\boxed{54}$

Grafy obou funkcí se protínají v bodě $\boxed{[4;54]}$.

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(12) Danou rovnici řešte v oboru reálných čísel:

(a) $\frac{2^{x+2}}{2^{1-x}}=\frac{\log 4}{\log 2}$ $\qquad$ Existuje vzorec: $\boxed{\log_r t=\frac{\log_{10} t}{\log_{10} r}}$

$\frac{\log 4}{\log 2}=\log_2 4=2$, neboť $2^2=4$ → lze vypočítat i na kalkulačce

$2^{x+2}\cdot 2^{x-1}=2$

$2^{2x+1}=2^1$

$2x+1=1$

$2x=0$ → $\boxed{x=0}$

Zkouška: $L: \frac{2^{0+2}}{2^{1-0}}=\frac{4}{2}=2$

$P=2$, $L=P$

b) $$\frac{3^{5x+2}}{3^{1-2x}} = \frac{\log 125}{\log 5}$$

Podle vzorce na str. 13 platí: $\frac{\log 125}{\log 5} = \log_5 125$, a se rovná 3, neboť $5^3 = 125$

$$3^{5x+2} \cdot 3^{2x-1} = 3$$

$$3^{7x+1} = 3^1$$

$$7x+1 = 1$$

$$7x = 0$$

$$\boxed{x=0}$$

Zkouška: bez užití pomoci kalkulačky

$$L = \frac{3^2}{3^1} = 3, \quad P = 3$$

---

c) $$\left(\frac{9}{25}\right)^{x+1} \cdot \left(\frac{5}{3}\right)^{2x} = \left(\frac{3}{5}\right)^{x}$$

$$\frac{9^{x+1}}{25^{x+1}} \cdot \frac{5^{2x}}{3^{2x}} = \frac{3^x}{5^x}$$

$$9^{x+1} \cdot 25^{-x-1} \cdot 5^{2x} \cdot 3^{-2x} = 3^x \cdot 5^{-x}$$

$$(3^2)^{x+1} \cdot (5^2)^{-x-1} \cdot 5^{2x} \cdot 3^{-2x} = 3^x \cdot 5^{-x}$$

$$3^{2x+2} \cdot 5^{-2x-2} \cdot 5^{2x} \cdot 3^{-2x} = 3^x \cdot 5^{-x}$$

$$3^2 \cdot 5^{-2} = 3^x \cdot 5^{-x}$$

$$\frac{9}{25} = \left(\frac{3}{5}\right)^x$$

$$\left(\frac{3}{5}\right)^x = \left(\frac{3}{5}\right)^2 \Rightarrow \boxed{x=2}$$

Zkouška: $L = \left(\frac{9}{25}\right)^3 \cdot \left(\frac{5}{3}\right)^4 = \frac{9}{25}$

$P = \left(\frac{3}{5}\right)^2 = \frac{9}{25}$; $L = P$

---

d) $$\left(\frac{7}{3}\right)^{1-3x} \cdot \frac{9}{49} = \left(\frac{49}{9}\right)^{2x+1}$$

$$\left(\frac{7}{3}\right)^{1-3x} \cdot \left(\frac{49}{9}\right)^{-1} = \left(\frac{7^2}{3^2}\right)^{2x+1}$$

$$\frac{7^{1-3x}}{3^{1-3x}} \cdot \left(\frac{7^2}{3^2}\right)^{-1} = \frac{7^{4x+2}}{3^{4x+2}}$$

$$7^{1-3x} \cdot 3^{3x-1} \cdot 7^{-2} \cdot 3^2 = 7^{4x+2} \cdot 3^{-4x-2}$$

$$7^{-3x-1} \cdot 3^{3x+1} = 7^{4x+2} \cdot 3^{-4x-2}$$

$$\frac{7^{-3x-1}}{7^{4x+2}} = \frac{3^{-4x-2}}{3^{3x+1}}$$

$$7^{-3x-1} \cdot 7^{-4x-2} = 3^{-4x-2} \cdot 3^{-3x-1}$$

$$7^{-7x-3} = 3^{-7x-3}$$

$7 = 3 \Rightarrow$

Daná rovnice $\boxed{\text{nemá řešení}}$

Konec tématu Funkce:

1.1 Funkce a jejich vlastnosti

1.2 Mocninné funkce

1.3 Exponenciální a log. funkce

1.4 Logaritmus a logaritmické výpočty

1.5 Logaritmické a exponenciální rovnice