Logaritmické rovnice řešené v r. 2008 ze sbírky Ivana Bušeka atd., mající ze zkoušek na VŠ

Př. 10.1/94 Řešte v R rovnici

$$\log (x^3+1) - \log 7 - \log x = \log (x+1) - \log 6$$

Pomocí vět o logaritmech lze psát:

*DOZE STRUČNĚ

$$\log \frac{x^3+1}{7} - \log x = \log \frac{x+1}{\cancel{\log} 6}$$

$$\log \frac{\frac{x^3+1}{7}}{x} = \log \frac{x+1}{6}$$

$$\log \frac{x^3+1}{7x} = \log \frac{x+1}{6}$$

$$\frac{x^3+1}{7x} = \frac{x+1}{6}$$

$$6(x^3+1) = 7x(x+1)$$

$$6.(x^3+1^3) = 7x(x+1)$$

Vzorec: $A^3+B^3=(A+B).(A^2-AB+B^2)$

$$6.\underline{(x+1)}.(x^2-x+1)-7x\underline{(x+1)}=$$

vytkneme

$$(x+1).[6.(x^2-x+1)-7x]=0$$

$$(x+1).(6x^2-6x+6-7x)=0$$

$$(x+1).(6x^2-13x+6)=0$$

Tato rovnice platí, právě když

$$x+1=0 \quad \vee \quad 6x^2-13x+6=0$$

$$x=-1 \qquad x_{1_2}=\frac{13\pm\sqrt{25}}{12}= \begin{cases} \frac{3}{2} \\ \frac{2}{3} \end{cases}$$

Zkouška viz nahoře

$$* \log (x^3+1)\cdot\frac{1}{7}\cdot\frac{1}{x}$$

Pro $x=-1$

$L=\log(-1+1)-\log 7-\log(-1)$. Logaritmus není definován pro záporná čísla, pro $x=-1$ není řešením dané rovnice.

Pro $x=\frac{3}{2}$

$$L=\log\left[\left(\tfrac{3}{2}\right)^3+1\right]:7:\frac{3}{2}=$$

$$=\log\frac{35}{8}\cdot\frac{1}{7}\cdot\frac{2}{3}=\log\frac{70}{168}=\log\frac{5}{12}$$

$$P=\log\left(\tfrac{3}{2}+1\right)\cdot\frac{1}{6}=\log\frac{5}{2}\cdot\frac{1}{6}=$$

$$=\log\frac{5}{12}\ ;\ L=P$$

Pro $x=\frac{2}{3}$ jde to obdobně

$$L=\log\left[\left(\tfrac{2}{3}\right)^3+1\right]\cdot\frac{1}{7}\cdot\frac{3}{2}=$$

$$=\log\frac{35}{27}\cdot\frac{1}{7}\cdot\frac{3}{2}=\log\frac{105}{378}=\frac{5}{18}$$

$$P=\log\left(\tfrac{2}{3}+1\right)\cdot\frac{1}{6}=\log\frac{5}{3}\cdot\frac{1}{6}=\frac{5}{18}$$

$L=P$

Př. 10.2/95 Řešte v R rovnici:

$$\log x+\log\sqrt{x}+\log\sqrt[4]{x}+\log\sqrt[8]{x}+\dots=2$$

$$\log x+\frac{1}{2}\log x+\frac{1}{4}\log x+\frac{1}{8}\log x+\dots=2$$

Pokračování

$$\underbrace{\left(1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots\right)}\cdot \log x = 2$$

je geometrická řada, pro jejíž součet s platí $s=\dfrac{a_1}{1-q}$, kde

$a_1 = 1,\ q=\frac{1}{2}$ $\quad \cdots\ S=\dfrac{1}{1-\frac{1}{2}}=\dfrac{1}{\frac{1}{2}}=2$

$$\left.\begin{array}{l} 2\cdot \log x = 2 \\ \log x = 1 \end{array}\right\}\ \log_{10} x = 1 \Rightarrow x = 10^1 \qquad \boxed{x=10}$$

Zkouška:

$L=\log 10+\log\sqrt{10}+\log\sqrt[4]{10}+\log\sqrt[8]{10}+\ldots=\log 10+\log 10^{\frac{1}{2}}+\log 10^{\frac{1}{4}}+\log 10^{\frac{1}{8}}\ldots \log 10+\frac{1}{2}\cdot\log 10+\frac{1}{4}\log 10+\frac{1}{8}\log 10+\cdots$

$=\log_{10} 10\left(1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots\right)=1.\ 2=2$, $P=2$; $L=P$

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* Př. 10.3 196 v R řešte pomocí základu

$$(\log_x 3)\cdot\left(\log_{\frac{x}{3}} 3\right)=\left(\log_{\frac{x^2}{9}} 3\right)$$

Použijeme vzorec: $\log_r t=\dfrac{\log t}{\log r}$

$$\frac{\log 3}{\log x}\cdot\frac{\log 3}{\log\frac{x}{3}}=\frac{\log 3}{\log\frac{x^2}{9}}\quad \Big|\cdot\frac{1}{\log 3}$$

Pozor: součin neurčíme tak, že vydělíme pouze 1 činitel.

$$\frac{1}{\log x}\cdot\frac{\log 3}{\log x-\log 3}=\frac{1}{\log x^2-\log 3^2}$$

$$\frac{\log 3}{\log x\cdot(\log x-\log 3)}=\frac{1}{2.\log x-2\log 3}$$

$$\frac{\log 3}{\log x\,(\log x-\log 3)}=\frac{1}{2(\log x-\log 3)}$$

Podle $\frac{2}{3}=\frac{4}{6}\ \ldots\ \frac{3}{2}=\frac{6}{4}$

$$\frac{\log x\,(\log x-\log 3)}{\log 3}=2.(\log x-\log 3)$$

$$\frac{\log x(\log x-\log 3)}{\log 3}-2.(\log x-\log 3)=0$$

$$\frac{\log x\,(\log x-\log 3)-2\log 3(\log x-\log 3)}{\log 3}=0 \qquad \text{Vytkneme}$$

$$\frac{(\log x-\log 3).(\log x-2.\log 3)}{\log 3}=0$$

$$(\log x-\log 3)\cdot\left(\frac{\log x}{\log 3}-\frac{2\cdot\cancel{\log 3}}{\cancel{\log 3}}\right)=0$$

$$(\log x-\log 3)\cdot\left(\frac{\log x}{\log 3}-2\right)=0 \Rightarrow$$

$$\log x-\log 3=0 \quad \vee \quad \frac{\log x}{\log 3}-2=0$$

$$\log x=\log 3 \qquad \frac{\log x}{\log 3}=2$$

$$\boxed{x=3}$$

$$\log x=2\cdot\log 3$$

$$\log x=\log 3^2$$

$$\log x=\log 9$$

$$\Rightarrow \boxed{x=9}$$

Zkouška:
Pro $x=3$ není $\log_{\frac{x}{3}} 3$ definován, $\log_1 3$ ... základ logaritmu je >1 a ne 1.

Pro $x=9$

$$L=\underbrace{\log_9 3}_{a}\cdot\underbrace{\log_3 3}_{b} \qquad P=\underbrace{\log_{\frac{81}{9}} 3}_{c}\cdot$$

$$\log_9 3=a \qquad \log_3 3=b \qquad \left(\frac{81}{9}\right)^c=3$$

$$9^a=3 \qquad 3^b=3^1 \qquad 9^c=3$$

$$(3^2)^a=3^1 \qquad b=1 \qquad 3^{2c}=3^1$$

$$3^{2a}=3^1 \qquad a\cdot b=\frac{1}{2}\cdot 1 \qquad 2c=1$$

$$a=\frac{1}{2} \qquad \boxed{\frac{1}{2}=L} \qquad c=\frac{1}{2} \qquad \boxed{P=\frac{1}{2}}$$

$$L=P$$

$x=9$ je řešením dané logar. rovnice.

Př. 10.4/98

$$\log x^{2\log\sqrt{x}}+\log\frac{1}{x^2}=3$$

$$\log x^{2\log\sqrt{x}}+\log x^{-2}=3$$

$$2\cdot\log\sqrt{x}\cdot\log x-2\log x=3$$

$$2\cdot\log x^{\frac{1}{2}}\cdot\log x-2\log x=3$$

$$\frac{1}{2}\cdot 2\log x\cdot\log x-2\log x-3=0$$

Substituce: $\log x=y$

$$y^2-2y-3=0$$

$$y_{1,2}=\frac{2\pm\sqrt{4+12}}{2}$$

$$y_{1,2}=\frac{2\pm 4}{2}=\begin{cases} y_1=3 \\ y_2=-1\end{cases}$$

Zpět: $\log x=3 \qquad \log x=-1$

$$\log_{10} x=3 \qquad \log_{10} x=-1$$

$$\boxed{x=10^3} \qquad \boxed{x=10^{-1}}$$

Zkouška potvrdí správnost

Př. 10.7/101 Řešte v R rovnici:

$$\log(x+1)+\log(x-1)-\log(x-2)=\log 8$$

$$\log\frac{(x+1)\cdot(x-1)}{x-2}=\log 8$$

$$\log\frac{x^2-1}{x-2}=\log 8$$

$$\frac{x^2-1}{x-2}=8$$

$$x^2-1=8x-16$$

$$x^2-8x+15=0$$

$$x_{1,2}=\frac{8\pm\sqrt{64-60}}{2}=\frac{8\pm 2}{2}=\begin{cases}\boxed{x_1=5}\\ \boxed{x_2=3}\end{cases}$$

Zkouška: pro $x=5$

$$L=\log 6+\log 4-\log 3-\log\frac{6\cdot 4}{3}=\log 8$$

$$P=\log 8 \ldots L=P$$

Zkouška pro $x=3$: $L = \log 4 + \log 2 - \log 1 = \log \frac{4 \cdot 2}{1} = \log 8$

$P = \log 8 \;\dots\; L = P$

Př. 10.8/101 Řešte v R rovnici:

$$\log(2x-3) + \log 3x = \log(8x-12)$$
$$\log(2x-3) \cdot 3x = \log(8x-12)$$
$$(2x-3) \cdot 3x = 8x-12$$
$$6x^2 - 9x - 8x + 12 = 0$$
$$6x^2 - 17x + 12 = 0$$
$$x_{1,2} = \frac{17 \pm \sqrt{36-36}}{12} = \frac{17}{12}$$

Zkouška:

$\log\left(2 \cdot \frac{17}{12} - 3\right) = -\frac{1}{6} \Rightarrow$

$-\frac{1}{6} \neq R \Rightarrow$ Rovnice nemá v R řešení

Př. 10.9/101 Řešte v R rovnici:

$$\log\sqrt{3x-5} + \log\sqrt{7x-3} = 1 + \log\frac{\sqrt{11}}{10}$$
$$\log(\sqrt{3x-5}) \cdot (\sqrt{7x-3}) = \log 10 + \log\frac{\sqrt{11}}{10}$$
$$\log\sqrt{(3x-5) \cdot (7x-3)} = \log 10 \cdot \frac{\sqrt{11}}{10}$$
$$\log\sqrt{21x^2 - 44x + 15} = \log\sqrt{11}$$
$$\sqrt{21x^2 - 44x + 15} = \sqrt{11}$$
$$21x^2 - 44x + 15 = 11$$
$$21x^2 - 44x + 4 = 0$$
$$x_{1,2} = \frac{44 \pm \sqrt{16\,000}}{42}$$
$$x_{1,2} = \frac{44 \pm 40}{42} \quad \boxed{x_1 = 2} \qquad x_2 = \frac{2}{21} \text{ ne}$$

Zkouška pro $x_2 = \frac{2}{21}$ ... $L = \log\sqrt{3 \cdot \frac{2}{21} - 5} = \sqrt{-\frac{33}{7}}$ nemá v R řešení

pro $x = 2$: $L = \log\sqrt{6-5} + \log\sqrt{11} = \log\sqrt{1} + \log\sqrt{11} = 0 + \log\sqrt{11} =$
$= \log\sqrt{11} \;\dots\; P = \log\sqrt{11} \;\dots\; L = P$

Př. 10.10/101 Řešte v R rovnici:

$$x^{3+4\log x} - 10x^6 = 0$$
$$x^3 \cdot x^{4\log x} = 10x^6, \text{ dosaď } \underline{x = 10^{\log x}}$$
$$(10^{\log x})^3 \cdot 10^{4\log x} = 10^1 \cdot (10^{\log x})^6$$
$$10^{3\log x} \cdot 10^{4\log x} = 10^1 \cdot 10^{6\log x}$$

Sub. $\log x = y$

$$10^{3y} \cdot 10^{4y} = 10^1 \cdot 10^{6y}$$
$$10^{7y} = 10^{1+6y}$$
$$7y = 1 + 6y$$

$y = 1$ zpět

$$\log x = 1$$
$$\log_{10} x = 1$$
$$x = 10^1$$
$$\boxed{x = 10}$$

Zkouška:

$L = 10^{3+4\cdot 1} - 10^1 \cdot 10^6 =$
$= 10^7 - 10^7 = 0,\ P = 0,\ L = P$

Poznámka: Ve sbírce je ještě další výsledek, a to $x = 10^{-\frac{1}{4}}$. K tomu jsem však nedospěl.

Př. 10.11/101 Řešte v $\mathbb{N}$

$$3\cdot 2^{\log x}+8\cdot 2^{-\log x}=5\left(1+10\log\sqrt[5]{100}\right)$$

$$3\cdot 2^{\log x}+8\cdot\frac{1}{2^{\log x}}=5\cdot\left(1+10\cdot\log 100^{\frac{1}{5}}\right)$$

$$3\cdot 2^{\log x}+8\cdot\frac{1}{2^{\log x}}=5+50\cdot\log\left(10^{\frac{2}{5}}\right)$$

1. substituce: $\log x = y$

$$3\cdot 2^{y}+8\cdot\frac{1}{2^{y}}=5+50\cdot\frac{2}{5}\underbrace{\log 10}_{1}\quad|\cdot 2^y$$

$$3\cdot 2^y 2^y+8=5+20\cdot 1\cdot 2^y$$

$$3\cdot 2^y 2^y+8-25z \qquad \text{II. sub. } 2^y=z$$

$$3z^2-25z+8=0$$

$$z_{1,2}=\frac{25\pm\sqrt{529}}{6}=\frac{25\pm 23}{6}=\begin{cases} z_1=8\\ z_2=3\end{cases}$$

Opět do II.

$$2^y=8 \qquad 2^y=\frac{1}{3}$$

nevyhovuje, neboť $x\in\mathbb{N}$ (viz zadání)

$$2^y=2^3$$

$$y=3$$

Zpět do I.

$$\log x=3$$

$$\log_{10}x=3$$

$$x=10^3\ (=1000)$$

Zkouška:

$$L=3\cdot 2^3+\frac{8}{2^3}=3\cdot 8+1=25$$

$$P=5\cdot\left(1+10\cdot\log 10^{\frac{2}{5}}\right)=$$

$$=5\cdot\left(1+10\cdot\frac{2}{5}\log 10\right)=5\left(1+4\cdot 1\right)=$$

$$=5\cdot 5=25\ ,\quad L=P$$

Př. 10.13/102 Řešte v $\mathbf{N}$ pomocí (R upraveno na N)

$$\log\left(2^{2\log x}\right)-\log 2^{3\sqrt{\log x}}=2\cdot\log 2$$

$$2\log x\cdot\cancel{\log 2}-3\sqrt{\log x}\cdot\cancel{\log 2}=2\cdot\cancel{\log 2}\quad|:\log 2$$

$$2\log x-3\sqrt{\log x}=2$$

$$2\log x-2=3\sqrt{\log x}$$

$$2(\log x-1)=3\sqrt{\log x}$$

Subst. $\log x=y$

$$2(y-1)=3\sqrt{y}$$

$$\sqrt{y}=\frac{2y-2}{3}$$

$$\left(\sqrt{y}\right)^2=\left(\frac{2y-2}{3}\right)^2$$

$$y=\frac{4y^2-8y+4}{9}$$

$$9y=4y^2-8y+4$$

$$4y^2-17y+4=0$$

$$y_{1,2}=\frac{17\pm\sqrt{225}}{8}$$

$$y_{1,2}=\frac{17\pm 15}{8}=\begin{cases} y_1=4\\ y_2=\frac{1}{4}\end{cases}$$

Opět:

$$\log x=4 \qquad \log x=\frac{1}{4}$$

$$\log_{10}x=4 \qquad \log_{10}x=\frac{1}{4}$$

$$\boxed{x=10^4} \qquad x=10^{\frac{1}{4}}$$

nevyhovuje

Zkouška pro $x=10^4$

$$L=\log\left(2^{2\cdot 4}\right)-\log\left(2^{3\sqrt{4}}\right)=\log 2^8-\log 2^6=$$

$$=\log\frac{2^8}{2^6}=\log 2^2=\log 4$$

$$P=2\cdot\log 2=\log 2^2=\log 4$$

$=$ L=P

Příklady na log. rovnice ze sbírky „Matem. minimum" od dr. Kubíčka a spol.

Př. 3.11 a/173 Určete x, jestliže platí:

$\log_4 x = 2 \cdot \log_4 5 - \frac{1}{2} \log_4 25 - 2$

$\log_4 x = 2 \log_4 5 - \frac{1}{2} \log_4 5^2 - 2$

$\log_4 x = 2 \log_4 5 - 2 \cdot \frac{1}{2} \log_4 5 - 2$

$\log_4 x = \underbrace{2 \log_4 5 - 1 \log_4 5} - 2$

$\log_4 x = \log_4 5 - 2 \quad \ldots \quad 4^a = -2 \quad \frac{1}{16} = \frac{1}{4^2} = 4^{-2}$

$= 4^{-2}$

$\downarrow$

$\log_4 x - \log_4 5 = -2$

$\log_4 \frac{x}{5} = \log_4 \frac{1}{16}$

$\frac{x}{5} = \frac{1}{16} \Rightarrow \boxed{x = \frac{5}{16}}$

Př. 3.11a/173 určit x

$\log x = \log a + \log b - \log c$

$\log x = \log \frac{ab}{c}$

$\boxed{x = \frac{ab}{c}}$; pro $a>0, b>0, c>0$

Př. 3.11b/173

$\log x = \log a - 2 \log b + \frac{1}{2} \log c$

$\log x = \log a + \log c^{\frac{1}{2}} - \log b^2$

$\log x = \log \frac{a \cdot c^{\frac{1}{2}}}{b^2}$

$\log x = \log \frac{a \cdot \sqrt{c}}{b^2}$

$\boxed{x = \frac{a \cdot \sqrt{c}}{b^2}}$

Př. 3.11c/173

$\log x = 2(\log 3 + \log 5) - \frac{1}{2} \log 9$

$\log x = 2 \cdot \log 3 + 2 \log 5 - \frac{1}{2} \log 9$

$\log x = \log 3^2 + \log 5^2 - \log 9^{\frac{1}{2}}$

$\log x = \log \frac{9 \cdot 25}{\sqrt{9}}$

$\log x = \log 75$

$\boxed{x = 75}$

Př. 4.2d/173

$\log_2 (4x-4) - \log_2 (3-x) = 2$

$\log_2 \frac{4x-4}{3-x} = 2 \quad \ldots \quad \log_2 y = 2$

$y = 2^2 \ldots y = 4$

$\log_2 \frac{4x-4}{3-x} = \log_2 4$

$\frac{4x-4}{3-x} = 4$

$4x - 4 = 12 - 4x$

$8x = 16$

$\boxed{x = 2}$

Zkouška:

$L = \log_2 (8-4) - \log_2 (3-2) =$

$\log_2 4 - \log_2 1 = 2 + 0 = 2,$

neboť $2^a = 4 \quad 2^b = 1$

$2^a = 2^2 \quad 2^b = 2^0$

$a = 2 \quad b = 0$

4.2e/73

$\log(2x+9) - 2\log x + \log(x-4) = 2 - \log 50$

$\log(2x+9) + \log(x-4) - \log x^2 = \log 100 - \log 50$

$\log \dfrac{(2x+9)\cdot(x-4)}{x^2} = \log \dfrac{100}{50}$

$\dfrac{(2x+9)\cdot(x-4)}{x^2} = 2$

$\cancel{2x^2} + x - 36 = \cancel{2x^2}$

$x - 36 = 0 \Rightarrow \boxed{x = 36}$

Zkouška:

$L = \log(72+9) - 2\log 36 + \log 32 =$

$= \log 81 - 2\cdot\log 36 + \log 32 \doteq 0{,}301\,029\,995$

$P = 2 - \log 50 \doteq 0{,}301\,029\,995$; $L = P$

Př. 4.3/73

$(\log_3 x)^2 - 3\log_3 x - 10 = 0$

Substituce: $\log_3 x = y$

$y^2 - 3y - 10 = 0$

$y_{1,2} = \dfrac{3 \pm \sqrt{9+40}}{2} = \begin{cases} y_1 = 5 \\ y_2 = -2 \end{cases}$

Řešení: $\log_3 x = 5$ $\quad \log_3 x = -2$

$x = 3^5$ $\quad x = 3^{-2}$

$\boxed{x = 243}$ $\boxed{x = \frac{1}{9}}$

Př. 4.3b/73

$2\log x = 3 + \dfrac{2}{\log x}$ ... Sub. $\log x = y$

$2y = 3 + \dfrac{2}{y} \quad /\cdot y$

$2y^2 = 3y + 2$

$2y^2 - 3y - 2 = 0$

$y_{1,2} = \dfrac{3 \pm \sqrt{9+16}}{4}$

$y_{1,2} = \dfrac{3 \pm 5}{4} = \begin{cases} y_1 = 2 \\ y_2 = -\frac{1}{2} \end{cases}$

Řešení: $\log x = 2$ $\quad \log x = -\frac{1}{2}$

$\log_{10} x = 2$ $\quad \log_{10} x = -\frac{1}{2}$

$x = 10^2$ $\quad x = 10^{-\frac{1}{2}}$

$\boxed{x = 100}$ $\quad x = \dfrac{1}{10^{\frac{1}{2}}} = \dfrac{1}{\sqrt{10}} = \dfrac{\sqrt{1}}{\sqrt{10}} = \sqrt{\dfrac{1}{10}} = \sqrt{0{,}1}$ ... $\boxed{x = \sqrt{0{,}1}}$

Zkouška: pro $x = 100$

$L = 2\cdot 2 = 4$, $P = 3 + \frac{2}{2} = 4 \Rightarrow L = P$

Zkouška pro $x = \sqrt{0{,}1}$

$L = 2\cdot\left(-\frac{1}{2}\right) = -1$, $P = 3 + \dfrac{\frac{2}{1}}{-\frac{1}{2}} = 3 - 4 = -1$

$L = P$

Př. 4.3c/73

$$1+\log x^3=\frac{10}{\log x}$$

$$1+3\log x=\frac{10}{\log x}$$

Sub. $\log x = y$

$$1+3y=\frac{10}{y} \quad |\cdot y$$

$$y+3y^2=10$$

$$3y^2+y-10=0$$

$$y_{1,2}=\frac{-1\pm\sqrt{121}}{6}$$

$$y_{1,2}=\frac{-1\pm 11}{6} \quad y_1=-2, \quad y_2=\frac{5}{3}$$

Zpět:

$\log_{10} x=-2$; $\log x=\frac{5}{3}$

$\boxed{x=10^{-2}}$ $\quad x=10^{\frac{5}{3}}$ $\quad \boxed{x=\sqrt[3]{10^5}}$

Zkouška pro $x=10^{-2}$

$$L=1+\log(10^{-2})^3=1+\log 10^{-6}=1-6\cdot\log 10=1-6\cdot 1=-5$$

$$P=\frac{10}{\log 10^{-2}}=\frac{10}{-2\log 10}=\frac{10}{-2\cdot 1}=\frac{10}{-2}=-5 \ldots L=P$$

Zkouška pro $x=10^{\frac{5}{3}}$

$$L=1+\log(10^{\frac{5}{3}})^3=1+\log 10^5=1+5\cdot\log 10=1+5\cdot 1=6$$

$$P=\frac{10}{\log_{10} 10^{\frac{5}{3}}}=\frac{10}{\frac{5}{3}\cdot\log 10}=\frac{10}{\frac{5}{3}\cdot 1}=\frac{\frac{10}{1}}{\frac{5}{3}}=\frac{30}{5}=6;\ L=P$$

Př. 4.3d/73

$$(2+\log x)\cdot\log x=-1$$

Substituce: $\log x=y$

$$(2+y)\cdot y=-1$$

$$2y+y^2+1=0$$

$$y^2+2y+1=0$$

$$y_{1,2}=\frac{-2\pm\sqrt{4-4}}{2}=\frac{-2}{2}=-1$$

Zpět: $\log x=-1$

$$\log_{10} x=\log\frac{1}{10}$$

$$\boxed{x=\frac{1}{10}}$$

Zkouška: $L=(2+\log\frac{1}{10})\cdot\log\frac{1}{10}=[2+(-1)]\cdot(-1)=$
$=(2-1)\cdot(-1)=1\cdot(-1)=-1$, $P=-1$, $L=P$

Příklad ze zkoušek na vysoké školy:

Vypočítej hodnotu (nejde o rovnici) výrazu:

$$\log_2\sqrt{2}-\log_2\sqrt[4]{2^3}+\log_2\sqrt[4]{2^5}=\log_2 2^{\frac{1}{2}}-\log_2 2^{\frac{3}{4}}+\log_2 2^{\frac{5}{4}}=$$

$$=\frac{1}{2}\log_2 2-\frac{3}{4}\log_2 2+\frac{5}{4}\log_2 2=\log_2 2\left(\frac{1}{2}-\frac{3}{4}+\frac{5}{4}\right)=\log_2 2\cdot 1=\log_2 2$$

Označme $\log_2 2=x \Rightarrow 2^x=2$

$$2^x=2^1 \Rightarrow x=1$$

$\boxed{\text{Výraz má hodnotu 1.}}$ (mohlo se psát i jako rovnice: $\log_2\sqrt{2}-$ atd $=x$)

Příklad ze zkoušek na vysoké školy: $\log_{16} 32=x$

$$16^x=32$$

$$(2^4)^x=2^5$$

$$2^{4x}=2^5$$

$$4x=5$$

$$\boxed{x=\frac{5}{4}}$$

Příklad ze zkoušek na vysoké školy:

$$\log_5 \sqrt{5} - \log_5 \sqrt[4]{5^3} + \log_5 \sqrt[4]{5^5} = x$$

$$\log_5 5^{\frac{1}{2}} - \log_5 5^{\frac{3}{4}} + \log_5 5^{\frac{5}{4}} = x$$

$$\frac{1}{2}\log_5 5 - \frac{3}{4}\log_5 5 + \frac{5}{4}\log_5 5 = x$$

$$\log_5 5 \cdot \left(\frac{1}{2} - \frac{3}{4} + \frac{5}{4}\right) = x$$

$$\log_5 5 \cdot 1 = x$$

$$\log_5 5 = x$$

$$5^x = 5$$

$$5^x = 5^1$$

$$\boxed{x = 1}$$

Zkouška pomocí výpočtu na kalkulačce s využitím vzorce $\log_r t = \frac{\log t}{\log r}$

$$L = \frac{\log\sqrt{5}}{\log 5} - \frac{\log\sqrt[4]{5^3}}{\log 5} + \frac{\log\sqrt[4]{5^5}}{\log 5} =$$

$$= \frac{\log 5^{\frac{1}{2}} - \log 5^{\frac{3}{4}} + \log 5^{\frac{5}{4}}}{\log 5} = 1; P = 1; L = P$$

Př. ze zkoušek na vysoké školy: Řešte rovnici:

$$2^{\log\frac{1}{2}x} = \frac{1}{4}$$

Subst.: $\log\frac{1}{2}x = y$

$$2^y = \frac{1}{4}$$

$$2^y = 2^{-2}$$

$$y = -2$$

$$\log\frac{1}{2}x = -2$$

$$\log\frac{x}{2} = -2$$

$$\log\frac{x}{2} = \log 0{,}01$$

$$\frac{x}{2} = \frac{1}{100}$$

$$x = \frac{2}{100}$$

$$\boxed{x = \frac{1}{50}}$$

NEBO $2^{\log\frac{1}{2}x} = 2^{-2}$

$$\log\frac{1}{2}x = \log\frac{1}{100}$$

$$\frac{1}{2}x = \frac{1}{100}$$

$$x = \frac{1}{50}$$

Zkouška: $L = 2^{\log\frac{1}{2}\cdot\frac{1}{50}} = 2^{\log\frac{1}{100}} = 2^{-2} = \frac{1}{4}$; $P = \frac{1}{4}$; $L = P$

Příklad ze zkoušek na vysoké školy: Řešte rovnici:

$5^{\log\frac{1}{2}x} = 25$ stačí řešit jednodušeji (oproti předchozímu příkladu):

$5^{\log\frac{1}{2}x} = 5^2 \Rightarrow \log\frac{1}{2}x = 2 \Rightarrow \log\frac{1}{2}x = \log 100$, čili

$$\frac{1}{2}x = 100$$

$$\frac{x}{2} = 100$$

$$\boxed{x = 200}$$

Zkouška: $L = 5^{\log\frac{1}{2}\cdot 200} = 5^{\log 100} = 5^2 = 25$

$P = 25$; $L = P$

Příklad ze zkoušek na vysoké školy

$$9^{\log\frac{1}{2}x} = \frac{1}{81}$$

$$9^{\log\frac{1}{2}x} = 9^{-2}$$

$$\log\frac{1}{2}x = -2$$

$$\log\frac{1}{2}x = \log\frac{1}{100}$$

$$\frac{1}{2}x = \frac{1}{100}$$

$$\boxed{x = \frac{1}{50}}$$

$$10^{\log\frac{1}{2}x} = \frac{1}{10}$$

$$10^{\log\frac{1}{2}x} = 10^{-1}$$

$$\log\frac{1}{2}x = -1$$

$$\log\frac{1}{2}x = \log\frac{1}{10}$$

$$\frac{1}{2}x = \frac{1}{10}$$

$$\boxed{x = \frac{1}{5}}$$

Vlastní příklad: $9^{\log \frac{1}{5}x} = \frac{1}{81}$

$9^{\log \frac{1}{5}x} = 9^{-2}$

$\log \frac{1}{5}x = -2 \rightarrow \log \frac{1}{5}x = \log \frac{1}{100}$

$\frac{1}{5}x = \frac{1}{100}$

$\boxed{x = \frac{1}{20}}$

Zkouška: $L = 9^{\log \frac{1}{5} \cdot \frac{1}{20}} = 9^{\log \frac{1}{100}} = 9^{-2} = \frac{1}{81}$; $P = \frac{1}{81}$

$L = P$

Příklad ze zkoušek na vysoké školy:

$\left(\frac{1}{2}\right)^{\log_2 x} = \frac{1}{4}$

$\left(2^{-1}\right)^{\log_2 x} = 2^{-2}$

$2^{-\log_2 x} = 2^{-2} \rightarrow -\log_2 x = -2$

$\log_2 x = 2$

$\log_2 x = \log_2 4$

$x = 4$

Zkouška: $L = \left(\frac{1}{2}\right)^{\log_2 4} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

$P = \frac{1}{4}$; $L = P$

Příklad ze zkoušek na vysoké školy:

$\left(\frac{1}{6}\right)^{\log_2 x} = 6^{-1}$

$\left(\frac{1}{6}\right)^{\log_2 x} = \left(\frac{1}{6}\right)^1 \rightarrow \log_2 x = 1$

$\log_2 x = \log_2 2$

$x = 2$

Zkouška: $L = \left(\frac{1}{6}\right)^{\log_2 2} = \left(\frac{1}{6}\right)^1 = \frac{1}{6} = 6^{-1}$

$P = -1$; $L = P$

Další příklady jsou ze sbírky příkladů z M k přijímacím zkouškám na VŠE

Určete základ z, je-li $\log_z \frac{27}{2} = -2$

$z^{-2} = \frac{27}{2}$

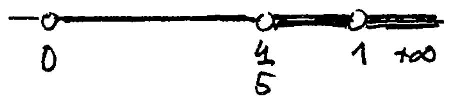
$z > 0$

$z^2 = \frac{2}{27}$

$|z| = \sqrt{\frac{2}{27}} = \frac{\sqrt{2}}{\sqrt{9 \cdot 3}} = \frac{1}{3} \cdot \sqrt{\frac{2}{3}} = \frac{1}{3} \cdot \frac{\sqrt{2} \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}} = \frac{1}{3} \cdot \frac{\sqrt{6}}{3} = \boxed{\frac{\sqrt{6}}{9}}$

V množině R řešme rovnici $\frac{2 \log x}{\log(5x-4)} = 1$

Nejdříve určíme D_f pro proměnnou x.

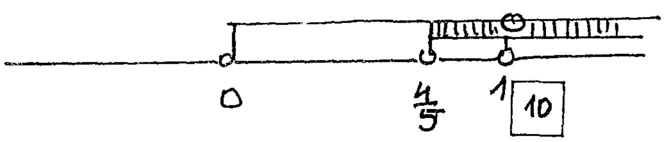


$x > 0 \wedge 5x - 4 > 0 \wedge \log(5x-4) \neq 0$

$x > 0 \wedge 5x > 4 \wedge 5x - 4 \neq 1$

$x \in \left(\frac{4}{5}; 1\right) \cup (1; \infty) = D_f$

$x > 0 \wedge x > \frac{4}{5} \wedge x \neq 1$



S podmínkou, že platí D_f řešíme:

$$\frac{2\log x}{\log(5x-4)} = 1$$

$$2\log x = \log(5x-4)$$

$$\log x^2 = \log(5x-4) \rightarrow x^2 = 5x-4$$

$$x^2 - 5x + 4 = 0$$

$$x_{1,2} = \frac{5\pm\sqrt{25-16}}{2} = \frac{5\pm\sqrt{9}}{2} = \frac{5\pm 3}{2} = \begin{cases} \boxed{x_1 = 4} \ \boxed{\text{je řešení!}} \\ x_2 = 1 \ (\text{je v rozporu s } D_f \end{cases}$$

Zkouška: $L = \dfrac{2\cdot\log 4}{\log(5\cdot 4-4)} = \dfrac{\log 4^2}{\log 16} = \dfrac{\log 16}{\log 16} = 1$; $P = 1$; $L = P$

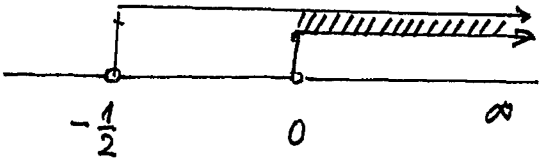
V množině R řešte:

$$\log\left(\tfrac{1}{2}+x\right) = \log\tfrac{1}{2} - \log x$$

Podmínka (D_f)

$\frac{1}{2}+x > 0 \wedge \frac{1}{2}+x \neq 0 \wedge x > 0$

$x > -\frac{1}{2} \wedge x \neq -\frac{1}{2} \wedge x > 0$



$x \in (0;\infty) = D_f$

$$\log\left(\tfrac{1}{2}+x\right) = \log\frac{\frac{1}{2}}{x}$$

$$\log\left(\tfrac{1}{2}+x\right) = \log\frac{1}{2x}$$

$$\tfrac{1}{2}+x = \frac{1}{2x} \quad |\cdot 2x$$

$$x + 2x^2 = 1$$

$$2x^2 + x - 1 = 0 \ \ldots \ x_{1,2} = \frac{-1\pm\sqrt{1+8}}{4} = \frac{-1\pm\sqrt{9}}{4} = \frac{-1\pm 3}{4} = \begin{cases} \boxed{x_1 = \tfrac{1}{2}} \ \boxed{\text{řešení}} \\ x_2 = -1 \ \text{nevyhovuje} \end{cases}$$

Zkouška: $L = \log\left(\frac{1}{2}+\frac{1}{2}\right) = \log 1 = 0 \ldots P = \log\frac{1}{2} - \log\frac{1}{2} = 0 \ldots L = P$

V množině R řešte:

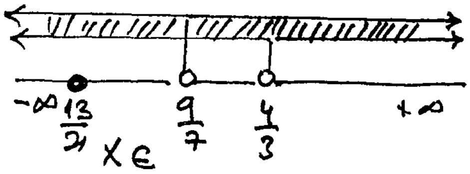
$$\log(3x-4)^2 + \log(7x-9)^2 = 2$$

Podmínka

$(3x-4)>0 \wedge (7x-9)>0 \vee (3x-4)<0 \wedge (7x-9)<$

$3x>4 \wedge 7x>9 \quad 3x<4 \wedge 7x<9$

$x>\frac{4}{3} \wedge x>\frac{9}{7} \vee x<\frac{4}{3} \wedge x<\frac{9}{7}$



$x \in$

$$2\cdot\log(3x-4) + 2\cdot\log(7x-9) = 2 \quad |:2$$

$$\log(3x-4) + \log(7x-9) = 1$$

$$\log(3x-4)\cdot(7x-9) = \log 10$$

$$(3x-4)\cdot(7x-9) = 10$$

$$21x^2 - 28x - 27x + 36 - 10 = 0$$

$$21x^2 - 55x + 26 = 0 \ \ldots \ x_{1,2} = \frac{55\pm\sqrt{841}}{42} = \frac{55\pm 29}{42} = \begin{cases} \frac{84}{42} = \boxed{2 = x_1} \\ \frac{26}{42} = \boxed{\frac{13}{21} = x_2} \end{cases}$$

Zkouška pro $x_1=2$ ... $L=\log(3\cdot 2-4)^2+\log(14-9)^2=\log 2^2+\log 5^2=$

$=\log 4+\log 25=\log 4\cdot 25=\log 100=2$; $P=2$; $L=P$

Zkouška pro $x_2=\frac{13}{21}$ ... $L=\log\left(3\cdot\frac{13}{21}-4\right)^2+\log\left(7\cdot\frac{13}{21}-9\right)^2=\log\frac{225}{49}+\log\frac{196}{9}=$

$\log\frac{225}{49}\cdot\frac{196}{9}=\log 100=2$; $P=2$; $L=P$

V množině R řešte:

$$\frac{\log 2x}{\log(4x-15)}=2$$

$$\log 2x=2\cdot\log(4x-15)$$

$$\log 2x=\log(4x-15)^2$$

$$2x=(4x-15)^2$$

$$2x=16x^2-120x+225$$

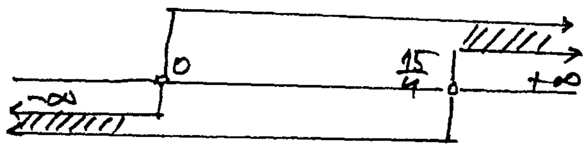
$$16x^2-122x+225=0$$

$$x_{1,2}=\frac{122\pm\sqrt{484}}{32}=\frac{122\pm 22}{32}= \begin{cases} x_1=\frac{9}{2} \\ x_2=\frac{15}{4}\ \text{(ne)} \end{cases}$$

Podmínka:

$(2x>0 \wedge 4x-15>0) \vee (2x<0 \wedge 4x-15<0)$

$\left(x>0 \wedge x>\frac{15}{4}\right) \vee \left(x<0 \wedge x<\frac{15}{4}\right)$



$x\in(-\infty;0)\cup\left(\frac{15}{4};\infty\right)=D_f$

Kořen $x_2=\frac{15}{4}$ je v rozporu s D_f. Řešení: $x_1=\frac{9}{2}$

Zkouška: $L=\dfrac{\log 2\cdot\frac{9}{2}}{\log\left(4\cdot\frac{9}{2}-15\right)}=\dfrac{\log 9}{\log 3}=$ na kalk. $=2$; $P=2$; $L=P$

V množině R řešte:

$$1+\log x^3=\frac{10}{\log x}$$

$$(1+3\cdot\log x)\cdot\log x=10$$

Substituce: $\log x=y$

$$(1+3y)\cdot y=10$$

$$y+3y^2-10=0$$

$$3y^2+y-10=0$$

$$y_{1,2}=\frac{-1\pm\sqrt{1+120}}{6}=\frac{-1\pm 11}{6}= \begin{cases} x_1=\frac{5}{3} \\ x_2=-2 \end{cases}$$

Řeš.: $\log x_1=\frac{5}{3}$ | $\log x_2=-2$

$x_1=10^{\frac{5}{3}}$ | $x_2=10^{-2}$

$\boxed{x_1=\sqrt[3]{10^5}}$ | $\boxed{x_2=\frac{1}{100}}$

Podmínka: $x>0$; $D_f\ (0;\infty)$

Zkouška: Pro x_1: $L = 1 + \log\left(10^{\frac{5}{3}}\right)^3 = 1 + \log 10^5 = 1 + 5 = 6$

$$P = \frac{10}{\log 10^{\frac{5}{3}}} = \frac{10}{\frac{5}{3}} = 6 \quad ; L = P$$

Pro x_2: $1 + \log\left(\frac{1}{100}\right)^3 = 1 + \log\left(10^{-2}\right)^3 = 1 + \log 10^{-6} = 1 - 6 = -5$

$$\frac{10}{\log\frac{1}{100}} = \frac{10}{\log 10^{-2}} = \frac{10}{-2} = -5 \quad ; L = P$$

⊛ Řešte:

$x^{\log x} = 1000x^2$ Rovnici nejdříve zlogaritmujeme.

$\log x^{\log x} = \log 1000x^2$

$\log x \cdot \log x = \log 1000 + \log x^2$

$\log x \cdot \log x = 3 + 2\log x$

Substituce: $\log x = y$

$y^2 = 3 + 2y$

$y^2 - 2y - 3 = 0$

$y_{1,2} = \frac{2 \pm \sqrt{4+12}}{2} = \begin{cases} y_1 = 3 \\ y_2 = -1 \end{cases}$

zpět: $\log x_1 = 3$ $\quad$ $\log x_2 = -1$

$\log x_1 = \log 1000$ $\quad$ $\log x_2 = \log\frac{1}{10}$

$\boxed{x_1 = 1000}$ $\quad$ $\boxed{x_2 = \frac{1}{10}}$

Zkouška:

pro $x_1 = 1000$

$L = 1000^3$; $P = 1000 \cdot 1000^2 = 1000^3$; $L = P$

pro $x_2 = \frac{1}{10}$

$L = \left(\frac{1}{10}\right)^{-1} = 10$; $P = 1000 \cdot \left(\frac{1}{10}\right)^2 = 1000 \cdot \frac{1}{100} = 10$; $L = P$

⊛ Řešte:

$\log_3 x + \log_3 (x+1) = 2 - \log_3 \frac{3}{2}$

Pomocný výpočet:

$\log_3 y = 2$

$y = 3^2 = 9$

$\log_3 x + \log_3 (x+1) = \log_3 9 - \log_3 \frac{3}{2}$

$\log_3 [x \cdot (x+1)] = \log \frac{\frac{9}{1}}{\frac{3}{2}}$

$\log_3 [x \cdot (x+1)] = \log 6$

$x^2 + x = 6$

$x^2 + x - 6 = 0$

$x_{1,2} = \frac{-1 \pm \sqrt{1+24}}{2} = \frac{-1 \pm 5}{2} = \begin{cases} \boxed{x_1 = 2} \\ x_2 = -3 \end{cases}$

nevyhovuje

Skouška: $L = \log_3 2 + \log_3(2+1) = \log_3 2 + \log_3 3 = \log_3(2\cdot3) = \log_3 6$

$P = 2 - \log_3 \frac{3}{2} = \log_3 9 - \log_3 \frac{3}{2} = \log_3 \frac{\frac{9}{1}}{\frac{3}{2}} = \log 6$; $L=P$

Řešte rovnici:

$\frac{1}{2}\log(3x+6) = \log(x-4)$

$\log(3x+6)^{\frac{1}{2}} = \log(x-4)$

$\sqrt{3x+6} = x-4$

$3x+6 = (x-4)^2$

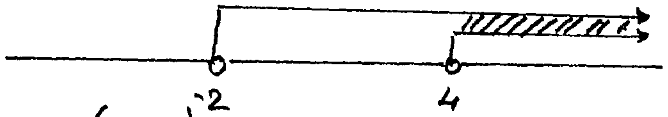
$3x+6 = x^2-8x+16$

$x^2 - 11x + 10 = 0$

$\rightarrow x_{1,2} = \frac{11\pm\sqrt{81}}{2} = \frac{11\pm 9}{2} = \boxed{x_1 = 10}$, $x_2 = 1$ (nevyhovuje)

Podmínka: $3x+6>0 \wedge x-4>0$

$x>-2 \wedge x>4$



$x \in (4;\infty)$

Skouška: $L = \frac{1}{2}\log 36 = \log(36)^{\frac{1}{2}} = \log\sqrt{36} = \log 6$

$P = \log(10-4) = \log 6 \ldots L=P$

(*) Řešte rovnici:

$\log(x+3)^2 - \log[4\cdot(x+1)^2] = 0$

$\log(x+3)^2 = \log 4 + \log(x+1)^2$

$\log(x+3)^2 - \log(x+1)^2 = \log 4$

$\log\frac{(x+3)^2}{(x+1)^2} = \log 4$

$\left(\frac{x+3}{x+1}\right)^2 = (\pm 2)^2$

$\frac{x+3}{x+1} = (\pm 2)$

$\rightarrow \frac{x+3}{x+1} = 2 \quad \vee \quad \frac{x+3}{x+1} = -2$

$x+3 = 2x+2 \qquad x+3 = -2x-2$

$\boxed{x_1 = 1} \qquad \boxed{x_2 = -\frac{5}{3}}$

Zkouška

pro $x_1=1$: $L = \log(1+3)^2 - \log[4(1+1)^2] = \log 16 - \log 16 = 0$

$P = 0$; $L=P$

pro $x_2 = -\frac{5}{5} \ldots L = \log(-\frac{5}{3}+3)^2 - \log[4\cdot(-\frac{5}{3}+1)^2] =$

$= \log(\frac{4}{3})^2 - \log[4\cdot(-\frac{2}{3})^2] = \log\frac{16}{9} - \log\frac{16}{9} = 0$

$P=0$; $L=P$

(**) $4^{\log_9 x^2} - 1 = 4^{1+\log_9 x} - 4^{-1+\log_9 x}$

$4^{\log_9 x^2} - 1 = 4\cdot4^{\log_9 x} - (\frac{1}{4}\cdot 4^{\log_9 x})$ ... vytkneme

$4^{y^2} - 1 = 4\cdot4^y - \frac{1}{4}\cdot4^y$

$4^{y^2} - 1 = 4^y\cdot(4-\frac{1}{4})$

Provedeme 1. substituci: $\log_9 x = y$

$4^{y^2}-1=4^y\cdot\frac{15}{4}$ .. Provedíme 2. substituci : $4^y=z$

$z^2-1=z\cdot\frac{15}{4}\ \ |\cdot 4$

$4z^2-4=15z$

$4z^2-15z-4=0$

$z_{1,2}=\frac{15\pm\sqrt{289}}{8}$

$z_{1,2}=\frac{15\pm 17}{8}=\begin{cases} z_1=4 \\ z_2=-\frac{1}{4}\end{cases}$

→ Opět do 2. sub.

$4^y=4$ $\quad$ $4^y=-\frac{1}{4}$ nemůže být

$4^y=4^1$ $\quad$ s ohledem na 1. sub.

$\boxed{y=4}$ Opět do 1. substit.

$\log_9 x=1$, čili

$\log_9 x=\log_9 9$, neboť $9^1=9$

$\boxed{x=9}$

Zkouška pro $x=9$ ... $L=4^{\log_9 81}-1=4^2-1=15$

$P=4^{1+\log_9 9}-4^{-1+\log_9 9}=4^{1+1}-4^{1-1}=4^2-4^0=15$

$L=P$

Šesté rovnice:

$\log_2\frac{1}{|x-1|-1}=1$

Rozlišme:

a) Pro $x-1>0$ je $|x-1|=x-1$

b) Pro $x-1<0$ je $|x-1|=-x+1$

Ad a)

$\log_2\frac{1}{x-1-1}=1$

$\log_2\frac{1}{x-2}=\log_2 2$

$\frac{1}{x-2}=2$

$2x-4=1$

$2x=5$

$\boxed{x_1=\frac{5}{2}}$

Ad 2)

$\log_2\frac{1}{-x+1-1}=1$

$\log_2\frac{1}{-x}=\log_2 2$

$-\frac{1}{x}=2$

$2x=-1$

$\boxed{x_2=-\frac{1}{2}}$

Zkoušky pro x_1:

$L=\log_2\frac{1}{|\frac{5}{2}-1|-1}=\log_2\frac{1}{|\frac{3}{2}|-1}=$

$=\log_2\frac{1}{\frac{3}{2}-1}=\log_2\frac{1}{\frac{1}{2}}=$

$=\log_2 2=1$, $P=1$; $L=P$

Zkouška pro x_2: $L=\log_2\frac{1}{|-\frac{1}{2}-1|-1}=\log_2\frac{1}{|-\frac{3}{2}|-1}=$

$=\log_2\frac{1}{\frac{3}{2}-1}=\log_2\frac{1}{\frac{1}{2}}=$

$=\log_2 2=1$, $P=1$, $L=P$

$\log_8\sqrt{3-x}+\log_8\sqrt{2x+18}=1$

$\log_8\left(\sqrt{3-x}\cdot\sqrt{2x+18}\right)=1$

$\log_8\sqrt{(3-x)\cdot(2x+18)}=1$

$\log_8\sqrt{-2x^2-12x+54}=\log_8 8$

$\sqrt{-2x^2-12x+54}=8$

$-2x^2-12x+54=64$

$x^2+6x+5=0$

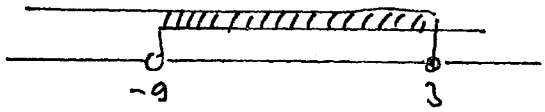
$x_{1,2}=\frac{-6\pm\sqrt{36-20}}{2}=\frac{-6\pm 4}{2}=$ $\boxed{x_1=-5}$; $\boxed{x_2=-1}$

Podm.

$3-x>0 \wedge 2x+18>0$

$-x>-3 \wedge 2x>-18$

$x<3 \wedge x>-9$



$D_f(-9;3)$

Zkouška pro x_1: $L=\log_8\sqrt{3+5}+\log_8\sqrt{-10+18}=\log_8\sqrt{8}+\log_8 8=$

$=2\cdot\log\sqrt{8}=2\cdot\log 8^{\frac{1}{2}}=\log_8 8^{\frac{1}{2}\cdot 2}=\log_8 8=1$

$P=1$; $L=P$

Zkouška pro x_2: $L=\log_8\sqrt{3+1}+\log_8\sqrt{-2+18}=\log_8\sqrt{4}+\log_8\sqrt{16}=$

$=\log_8 2+\log_8 4=\log_8 2\cdot 4=\log_8 8=1$; $P=1$; $L=P$

Řešte rovnici:

$\left[2\cdot(\log x)^3-\frac{3}{2}\log x\right]\cdot\log x=\log\sqrt{10}$ … Pomocný výpočet:

$\log_{10}\sqrt{10}=\log_{10}10^{\frac{1}{2}}=\frac{1}{2}$

1. substituce: $\log x=y$

$\left(2y^3-\frac{3}{2}y\right)\cdot y=\frac{1}{2}$

$2y^4-\frac{3}{2}y^2-\frac{1}{2}=0 \quad |\cdot 2$

$4y^4-3y^2-1=0$

2. sub. $y^2=z$

$4z^2-3z-1=0$

$z_{1,2}=\frac{3\pm\sqrt{25}}{8}$

$z_{1,2}=\frac{3\pm 5}{8}=$ $z_1=1$; $z_2=-\frac{1}{4}$ nevyhovuje

Zpět do 2. sub. $y^2=1$; $y_1=1$; $y_2=-1$

Zpět do 1. sub.

$\log_{10}x=1$ $\qquad \log_{10}x=-1$

$\boxed{x_1=10}$ $\qquad \boxed{x=\frac{1}{10}}$

Zkouška: pro x_1: ... $L=\left[2\cdot(\log 10)^3-\frac{3}{2}\cdot\log 10\right]\cdot\log 10=(2\cdot 1^3-\frac{3}{2}\cdot 1)\cdot 1=$

$=\frac{\cancel{2}}{3}(2-\frac{3}{2})\cdot 1=\frac{1}{2}\cdot 1=\frac{1}{2}$; $P=\log\sqrt{10}=\frac{1}{2}$; $L=P$

pro x_2 ... $L=\left[2\cdot(\log\frac{1}{10})^3-\frac{3}{2}\cdot\log\frac{1}{10}\right]\cdot\log\frac{1}{10}=$

$=\left[2\cdot(-1)^3-\frac{3}{2}\cdot(-1)\right]\cdot(-1)=(-2+\frac{3}{2})\cdot(-1)=-\frac{1}{2}\cdot(-1)=$

$=+\frac{1}{2}$; $P=\log\sqrt{10}=\frac{1}{2}$; $L=P$

Řešte rovnici:

$\log(2x-3)+\log 3x=\log(8x-12)$

$\log(2x-3)\cdot 3x=\log(8x-12)$

$(2x-3)\cdot 3x=8x-12$

$6x^2-9x=8x-12$

$6x^2-17x+12=0$

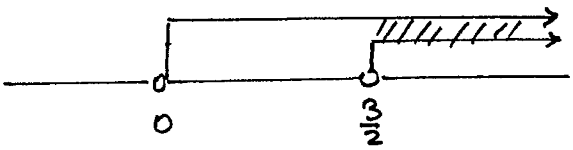
$x_{1,2}=\frac{17\pm\sqrt{1}}{12}=\frac{17\pm 1}{12}=$ $x_1=\frac{3}{2}$; $x_2=\frac{4}{3}\,(\doteq 1{,}3)$

Podmínka:

$2x-3>0 \wedge 8x-12>0 \wedge x>0$

$2x>3 \wedge 8x>12 \wedge x>0$

$x>\frac{3}{2} \wedge x>\frac{3}{2} \wedge x>0$



$x\in(\frac{3}{2};\infty)=D_f$

Rovnice nemá řešení ; $x_1, x_2 \notin D_f$ (je ve shodě s výsledky ve Sb.)

Řešte rovnici:

$\log 15x^2+\log 0{,}6x=\log 81^2$

$\log(15x^2\cdot 0{,}6x)=\log 81^2$

$\log 9x^3=\log 81$

$9x^3=9^4 \quad |:9$

$x^3=9^3$

$x=\sqrt[3]{9^3}$

$\boxed{x=9}$

Zkouška: $L=\log 15\cdot 9^2+\log 0{,}6\cdot 9=$

$=\log \underbrace{15\cdot 9^2\cdot 0{,}6}\cdot 9=\log 9\cdot 9^2\cdot 9=9^4$

$P=81^2=(9^2)^2=9^4$; $L=P$

Řešte rovnici:

$\log(2x+9)-2\log x+\log(x-4)=2-\log 50$

spojit

$$\log(2x+9)\cdot(x-4)-\log x^2=\log 100-\log 50$$

$$\log\frac{(2x+9)\cdot(x-4)}{x^2}=\log\frac{100}{50}$$

$$\frac{2x^2+9x-8x-36}{x^2}=2$$

$$\cancel{2x^2}+x-36=\cancel{2x^2}$$

$$\boxed{x=36}$$

Zkouška:

$L=\log(72+9)-2\cdot\log 36+\log(36-4)$

$=\log 81-\log 36^2+\log 32=$

$=\log\frac{81\cdot 32}{36^2}=\log 2$

$P=2-\log 50=\log 100-\log 50=\log 2$

$L=P$

Řešte rovnici:

$$\frac{\log x}{1-\log 2}=2$$

$$\frac{\log x}{\log 10-\log 2}=2$$

$$\frac{\log x}{\log\frac{10}{2}}=2$$

$$\frac{\log x}{\log 5}=2$$

$$\log x=2\cdot\log 5$$

$$\log x=\log 5^2$$

$$\boxed{x=25}$$

Zkouška: $L=\frac{\log 25}{1-\log 2}=$

$=\frac{\log 25}{\log 10-\log 2}=\frac{\log 25}{\log\frac{10}{2}}=$

$=\frac{\log 25}{\log 5}$ na kalk. $=2$

$P=2$; $L=P$

Řešte rovnici:

$$x^{\log x}=\sqrt[4]{10}$$

Obě strany rovnice zlogaritmujeme:

$$\log\left(x^{\log x}\right)=\log 10^{\frac{1}{4}}$$

$$\log x\cdot\log x=\frac{1}{4}$$

Substituce: $\log x=y$

$$y^2=\frac{1}{4}$$

$$y_{1,2}=\pm\frac{1}{2}=\begin{cases}y_1=\frac{1}{2}\\ y_2=-\frac{1}{2}\end{cases}$$

Zpět: $\log x_1=\frac{1}{2}$

$$\log x_1=\log 10^{\frac{1}{2}}$$

$$\boxed{x_1=\sqrt{10}}$$

$$\log x_2=\log 10^{-\frac{1}{2}}$$

$$x_2=10^{-\frac{1}{2}}$$

$$x_2=\frac{1}{10^{\frac{1}{2}}}$$

$$\boxed{x_2=\frac{1}{\sqrt{10}}}$$

Zkouška pro x_1: $L=\sqrt{10}^{\log\sqrt{10}}=\left(10^{\frac{1}{2}}\right)^{\log 10^{\frac{1}{2}}}=\left(10^{\frac{1}{2}}\right)^{\frac{1}{2}}=$

$=10^{\frac{1}{4}}=\sqrt[4]{10}$, $P=\sqrt[4]{10}$; $L=P$ ... pro x_2: $L=\left(\frac{1}{\sqrt{10}}\right)^{\log\frac{1}{\sqrt{10}}}=\left(10^{-\frac{1}{2}}\right)^{-\frac{1}{2}}=10^{\frac{1}{4}}=$

$=\sqrt[4]{10}$; $P=\sqrt[4]{10}$; $L=P$

⊗ Řešte: $x^{\log\sqrt[3]{x}}=\cancel{100}\,1000$ Provedeme zlogaritmování obou stran r.

$$\log\left(x^{\log\sqrt[3]{x}}\right) = \log 1000$$
$$\log\sqrt[3]{x}\cdot\log x = \log 1000$$
$$\log x^{\frac{1}{3}}\cdot\log x = \log 1000$$
$$\frac{1}{3}\cdot\underbrace{\log x}_{y}\cdot\underbrace{\log x}_{y} = \log 1000$$
$$\frac{1}{3}y^2 = 3$$
$$y^2 = 9$$
$$y_{1,2} = \pm 3$$

Zpět

$\log x = 3 \qquad \log x = -3$

$\log x = \log 1000 \qquad \log x = \log\frac{1}{1000}$

$x_1 = 1000 \qquad x_2 = \frac{1}{1000}$

Zkouška pro x_1:

$L = 1000^{\log\sqrt[3]{1000}} = 1000^{\log 10} = 1000^1 = 1000$

$P = 1000$, $L = P$

Zkouška pro x_2: $L = \left(\frac{1}{1000}\right)^{\log\sqrt[3]{\frac{1}{1000}}} = \left(\frac{1}{1000}\right)^{\log\frac{1}{10}} = \left(\frac{1}{1000}\right)^{-1} = 1000$

$P = 1000$; $L = P$

Řešte rovnici:

$$3^{\log 10x} = 81$$

obě strany zlogaritmujeme

$$\log 3^{\log 10x} = \log 81$$
$$\log 10x\cdot\log 3 = \log 81$$
$$\log 10x = \frac{\log 81}{\log 3}$$
$$\log 10 + \log x = \frac{\log 3^4}{\log 3}$$

$$1 + \log x = 4\cdot\frac{\log 3}{\log 3}$$
$$1 + \log x = 4\cdot 1$$
$$\log x = 3$$
$$\log x = \log 1000$$
$$\boxed{x = 1000}$$

Zkouška: $L = 3^{\log 10\,1000} = 3^{\log 10\,000} = 3^4 = 81$

$P = 81$; $L = P$

Řešte rovnici:

$$100^{\log(x+20)} = 10\,000$$

obě str. zlogaritmujeme

$$\log 100^{\log(x+20)} = \log 10\,000$$
$$\log(x+20)\cdot\log 100 = \log 10\,000$$
$$\log(x+20)\cdot 2 = 4$$
$$\log(x+20) = 2$$

Substituce: $x + 20 = y$

$\log_{10} y = 2 \Rightarrow y = 10^2 = 100$

Zpět: $x + 20 = 100$

$$\boxed{x = 80}$$

Zkouška: $L = 100^{\log(80+20)} = 100^{\log 100} = 100^2 = 10\,000$

$P = 10000$; $L = P$

V množině R řešte rovnici:

$\log_x(x^2-2x+2)=1 \Rightarrow x^1 = x^2-2x+2$

$x^2-3x+2=0$

$x_{1/2}=\frac{3\pm\sqrt{9-8}}{2}=\frac{3\pm 1}{2}$ ⟨ $x_1=2$, $x_2=1$

Zkouška pro $x_1=2$:

$L=\log_2(4-4+2)=\log_2 2=1$

$P=1$; $L=P$

Kořen x_2 jako základ logaritmu se nerovná číslu 1 ; tento kořen rovnici nevyhovuje

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Řešte v R:

$$\frac{1}{\log x+1}+\frac{6}{\log x+5}=1$$

Substituce: $\log x = y$

$$\frac{1}{y+1}+\frac{6}{y+5}=1$$

$$\frac{y+5+6y+6}{(y+1)\cdot(y+5)}=1$$

$$\frac{7y+11}{y^2+y+5y+5}=1$$

$$7y+11=y^2+6y+5$$

$$y^2-y-6=0$$

$y_{1,2}=\frac{1\pm\sqrt{1+24}}{2}=\frac{1\pm 5}{2}$ ⟨ $y_1=3$, $y_2=-2$

Pak:

$\log x = 3$

$\log x = \log 1000$

$\boxed{x_1=1000}$

$\log x = -2$

$\log x = \log\frac{1}{100}$

$\boxed{x_2=\frac{1}{100}}$

Zkouška pro x_1: $L=\frac{1}{\log 1000+1}+\frac{6}{\log 1000+5}=$

$=\frac{1}{3+1}+\frac{6}{3+5}=\frac{1}{4}+\frac{6}{8}=1$; $P=1$; $L=P$

pro x_2: $L=\frac{1}{\log\frac{1}{100}+1}+\frac{6}{\log\frac{1}{100}+5}=\frac{1}{-2+1}+\frac{6}{-2+5}=$

$=-1+2=1$; $P=1$; $L=P$

Řešte v R:

$$\log_2\frac{x}{4}=\frac{15}{\log_2\frac{x}{8}-1}$$

Pomocný výpočet: $2^x=4$, $2^x=2^2$, $x=2$; $2^x=8$, $2^x=2^3$, $x=3$

$$\log_2 x-\log_2 4=\frac{15}{\log_2 x-\log_2 8-1}$$

$$\log_2 x-2=\frac{15}{\log_2 x-3-1}$$

$$\log_2 x-2=\frac{15}{\log_2 x-4}$$

Substituce: $\log_2 x = y$

$$y-2=\frac{15}{y-4}$$

$$(y-2)\cdot(y-4)=15$$

$$y^2-6y+8-15=0$$

$$y^2-6y-7=0$$

$y_{1,2}=\frac{6\pm\sqrt{36+28}}{2}$

$y_{1,2}=\frac{6\pm 8}{2}$

$y_1=7$, $y_2=-1$

$\log_2 x = 7 \Rightarrow \boxed{x_1 = 2^7}$ $\quad \log_2 x = -1 \Rightarrow x = 2^{-1} = \boxed{\frac{1}{2} = x_2}$

Zkouška: pro x_1: $L = \log_2 \frac{2^7}{2^2} = \log_2 32$ $\quad P = \frac{15}{\log_2 \frac{2^7}{2^3} - 1} = \frac{15}{\log_2 2^4 - 1} = \frac{15}{4-1} = 5$

$= \log_2 2^5 = 5$ $\quad L = P$

Zkouška: pro x_2: $L = \frac{\frac{1}{2}}{4} = \log_2 \frac{1}{8} = \log_2 8^{-3} = -3$

$P = \frac{15}{\log \frac{\frac{1}{2}}{8} - 1} = \frac{15}{\log_2 \frac{1}{16} - 1} = \frac{15}{\log_2 16^{-1} - 1} = \frac{15}{\log_2 2^{-4} - 1} = \frac{15}{-5} = -3$ $\quad L = P$

Řešte rovnici:

$x^{3 + 4\log x} - 10x^6 = 0$

$x^3 \cdot x^{4\log x} - 10x^6 = 0$

$x^{4\log x} \cdot x^3 = 10x^6$

Zlogaritmujeme

$\log(x^{4\log x} \cdot x^3) = \log(10x^6)$

$\log x^{4\log x} + \log x^3 = \log 10 + \log x^6$

$4\log x \cdot \log x + 3 \cdot \log x = 1 + 6 \cdot \log x$

Substituce: $\log x = y$

$4y^2 + 3y = 1 + 6y$

$4y^2 - 3y - 1 = 0$

$y_{1,2} = \frac{3 \pm \sqrt{9+16}}{8} = \frac{3 \pm 5}{8}$ $\quad y_1 = 1$, $\quad y_2 = -\frac{1}{4}$

Opět:

$\log x = 1$ $\quad \log x_2 = -\frac{1}{4}$; $x_2 = 10^{-\frac{1}{4}}$

$\log x = \log 10$ $\quad \log x_2 = \log 10^{-\frac{1}{4}}$

$\boxed{x_1 = 10}$ $\quad \boxed{x_2 = 10^{-\frac{1}{4}}}$

Zkouška pro x_1:

$L = 10^{3 + 4\log 10} - 10 \cdot 10^6 =$

$= 10^{3+4} - 10^7 = 10^7 - 10^7 = 0$; $P = 0$; $L = P$

Zkouška pro x_2:

$L = (10^{-\frac{1}{4}})^{3 + 4\cdot(-\frac{1}{4})} - 10^1 \cdot (10^{-\frac{1}{4}})^6 =$

$= (10^{-\frac{1}{4}})^2 - 10^1 \cdot 10^{-\frac{6}{4}} = 10^{-\frac{1}{2}} - 10^{-\frac{1}{2}} = 0$

$P = 0$, $L = P$

Řešte: $\log_2(25^{x+3} - 1) = 2 + \log_2(5^{x+3} + 1)$

$\log_2(25^{x+3} - 1) - \log_2(5^{x+3} + 1) = 2$

$2^a = 4$

$2^a = 2^2$

$a = 2$

$\log_2 \frac{25^{x+3} - 1}{5^{x+3} + 1} \quad \log_2 4$

$$\frac{25^{x+3}-1}{5^{x+3}+1}=4$$

$$25^{x+3}-1=4\cdot(5^{x+3}+1)$$

$$25^x\cdot 25^3-1=4\cdot(5^x\cdot 5^3+1)$$

$$(5^x)^2\cdot 25^3-1=4\cdot 5^3\cdot 5^x+4$$

Subst. $5^x=y$

$$y^2\cdot 25^3-1=4\cdot 5^3 y+4$$

$$25^3y^2-500y-5=0 \quad |:5$$

$$3125y^2-100y-1=0$$

$$y_{1,2}=\frac{100\pm\sqrt{22500}}{6250}=\frac{100\pm 150}{6250}= \begin{cases} y_1=\frac{1}{25} \\ y_2=-\frac{1}{125}\end{cases}$$

Zpět: $5^x=\frac{1}{25}$ $\quad 5^x=-5^{-2}$

$5^x=5^{-2}$ $\quad$ nemá řešení

$\boxed{x_1=-2}$

Zkouška: $L=\log_2(25^{-2+3}-1)\log_2(25-1)=$ $\log_2 24$

$P=2+\log_2(5^{-2+3}+1)=\log_2 4+\log_2 6=$ $=\log_2 24$; $L=P$

Řešte v R:

$$\log_2(4^x+4)=\log_2 2^x+\log_2(2^{x+1}-3)$$

$$\log_2(4^x+4)-\log_2(2^{x+1}-3)=\log_2 2^x$$

$$\log_2\frac{4^x+4}{2^{x+1}-3}=\log_2 2^x$$

$$\frac{4^x+4}{2^{x+1}-3}=2^x$$

$$(2^2)^x+4=2^x\cdot(2^{x+1}-3)$$

$$(2^x)^2+4=2^x\cdot(2^x\cdot 2-3)$$

Subst. $2^x=y$

$$y^2+4=y\cdot(y\cdot 2-3)$$

$$y^2+4=2y^2-3y$$

$$y^2-3y-4=0$$

$$y_{1,2}=\frac{3\pm\sqrt{9+16}}{2}=\frac{3\pm\sqrt{25}}{2}=$$

$$=\frac{3\pm 5}{2}= \begin{cases} y_1=4 \\ y_2=-1\end{cases}$$

Zpět: $2^x=4$ $\quad 2^x=-1$

$2^x=2^2$ $\quad$ nevyhovuje

$\boxed{x=2}$

Zkouška:

$L=\log_2(4^2+4)=\log_2 20$

$P=\log_2 2^2+\log_2(2^3-3)=$

$=\log_2 4+\log_2 5=\log_2 4\cdot 5=$

$\log_2 20$

$L=P$

Řešte v R:

$$\log_2(9^{x-2}+7)=2+\log_2(3^{x-2}+1)$$

$$\log_2(9^{x-2}+7)-\log_2(3^{x-2}+1)=2$$

$$\log_2\frac{9^{x-2}+7}{3^{x-2}+1}=\log_2 4$$

$$\frac{9^{x-2}+7}{3^{x-2}+1}=4$$

$$9^{x-2}+7=4\cdot(3^{x-2}+1)$$

$$9^x\cdot 9^{-2}+7=4\cdot(3^x\cdot 3^{-2}+1)$$

$$(3^x)^2\cdot\frac{1}{81}+7=4\cdot(3^x\cdot\frac{1}{9}+1)$$

Substituce: $3^x=y$

$$y^2\cdot\frac{1}{81}+7=y\cdot\frac{4}{9}+4 \quad /\cdot 81$$

$$y^2+567=36y+324$$

$$y^2-36y+243=0$$

$$y_{1,2}=\frac{36\pm\sqrt{324}}{2}=\frac{36\pm 18}{2}=\begin{cases} y_1=27 \\ y_2=9\end{cases}$$

Zpět: $3^x=27$ $\quad$ $3^x=9$

$3^x=3^3$ $\quad$ $3^x=3^2$

$\boxed{x_1=3}$ $\quad$ $\boxed{x_2=2}$

Zkouška: pro $x_1=3$: $L=\log_2(9^1+7)=\log_2 16=4$

$P=2+\log_2(3+1)=2+\log_2 4=2+2=4$ $\}\ L=P$

pro $x_2=2$: $L=\log_2(9^0+7)=\log_2 8=3$

$P=2+\log_2(3^0+1)=2+\log_2 2=2+1=3$ $\}\ L=P$

Řešte v R

$$\log 2+\log(4^{-x-1}+9)=1+\log(2^{-x-1}+1)$$

$$\log(4^{-x-1}+9)-\log(2^{-x-1}+1)=\log 10-\log 2$$

$$\log\frac{4^{-x-1}+9}{2^{-x-1}+1}=\log\frac{10}{2}$$

$$\frac{4^{-x-1}+9}{2^{-x-1}+1}=5$$

$$4^{-x-1}+9=5\cdot(2^{-x-1}+1)$$

$$\frac{1}{4^{x+1}}+9=5\cdot\frac{1}{2^{x+1}}+5$$

1. substituce: $x+1=y$

$$\frac{1}{4^y}+9=\frac{5}{2^y}+5$$

$$\frac{1}{(2^y)^2}+9=\frac{5}{2^y}+5$$

2. substituce: $2^y=z$

$$\frac{1}{z^2}+9=\frac{5}{z}+5 \quad / \cdot z^2$$

$$1+9z^2=5z+5z^2$$

$$4z^2-5z+1=0$$

$$z_{1,2}=\frac{5\pm\sqrt{25-16}}{8}=$$

$$=\frac{5\pm3}{8}= \begin{cases} z_1=-\frac{1}{4} \\ z_2=1 \end{cases}$$

Zpět do sub. 2

$$2^y=-\frac{1}{4} \qquad 2^y=1$$

$$2^y=2^{-2} \qquad 2^y=2^0$$

$$y=-2 \qquad y=0$$

Zpět do sub. 1

$$x+1=-2 \;,\; x+1=0$$

$$\boxed{x_1=-3} \qquad \boxed{x_2=-1}$$

Zkouška pro x_1: $L=\log 2+\log(4^{2}+9)=\log 2+\log 25=\log 2.25=$
$=\log 50$

$P=1+\log(2^{2}+1)=\log 10+\log 5=\log 10.5=\log 50$

$L=P$

Zkouška pro x_2: $L=\log 2+\log(4^{0}+9)=\log 2.\log 10=\log 2.10=\log 20$

$P=1+\log(2^{0}+1)=\log 10+\log 2=\log 10.2=\log 20$

$L=P$