24a) **EXPONENCIÁLNÍ ROVNICE**

Za exponenciální rovnici budeme považovat takovou rovnici, která má vztah k exponenciální funkci. Proměnná (neznámá) $x \in R$ je v exponentu. Žádný mat.-text tuto rovnici přesně nedefinuje.

Řešte exponenciální rovnice (u některých proveďte zkoušku)

$2^x = 8$

$2^x = 2^3$

$\boxed{x = 3}$

$L = 2^3 = 8$

$P = 8$

$L = P$

$3^x = \frac{1}{81}$

$3^x = \frac{1}{3^4}$

$3^x = 3^{-4}$

$\boxed{x = -4}$

$3^{3x-1} = 1$

$3^{3x-1} = 3^0$

$3x - 1 = 0$

$3x = 1$

$\boxed{x = \frac{1}{3}}$

$81^{-x} = 27$

$(3^4)^{-x} = 3^3$

$3^{-4x} = 3^3$

$-4x = 3$

$\boxed{x = -\frac{3}{4}}$

$0{,}4^{3-2x} = 1$

$0{,}4^{3-2x} = 0{,}4^0$

$3 - 2x = 0$

$-2x = -3$

$\boxed{x = \frac{3}{2}}$

$\left(\frac{3}{5}\right)^{2x-5} = \left(\frac{5}{3}\right)^{-3}$

$\left(\frac{3}{5}\right)^{2x-5} = \left(\frac{3}{5}\right)^{3}$

$2x - 5 = 3$

$2x = 8$

$\boxed{x = 4}$

$5^{x+1} + 5^{x+2} = 750$

$5^x \cdot 5 + 5^x \cdot 25 = 750$

$5^x \cdot (5 + 25) = 750$

$5^x = 25$

$5^x = 5^2$

$\boxed{x = 2}$

$L = 5^3 + 5^4 = 750$

$P = 750$

$L = P$

$\left(\frac{3}{4}\right)^{\frac{x}{3}} = \sqrt[3]{\left(\frac{4}{3}\right)^x}$

$\left(\frac{3}{4}\right)^{\frac{x}{3}} = \left(\frac{4}{3}\right)^{\frac{x}{3}}$

$\left(\frac{3}{4}\right)^{\frac{x}{3}} = \left(\frac{3}{4}\right)^{-\frac{x}{3}}$

$\frac{x}{3} = -\frac{x}{3} \quad | \cdot 3$

$x = -x$

$2x = 0$

$\boxed{x = 0}$

$0{,}01^x = 10\,000$

$\left(\frac{1}{100}\right)^x = 10^4$

$\left(\frac{1}{10^2}\right)^x = 10^4$

$(10^{-2})^x = 10^4$

$10^{-2x} = 10^4$

$-2x = 4$

$\boxed{x = -2}$

$2^b + 2^{b+1} = 24$

$2^b + 2^b \cdot 2 = 24$

$2^b \cdot (1 + 2) = 24$

$2^b = 8$

$2^b = 2^3$

$\boxed{b = 3}$

$25^x = \left(\frac{1}{5}\right)^{x^2}$

$(5^2)^x = (5^{-1})^{x^2}$

$5^{2x} = 5^{-x^2}$

$2x = -x^2 \quad | : x$

$\boxed{x = -2}$

$2^x \cdot 5^x = \left[0{,}1 \cdot (10^{x-1})\right]^5$

$(2 \cdot 5)^x = \left(\frac{1}{10}\right)^5 \cdot 10^{5x-5}$

$10^x = 10^{-5} \cdot 10^{5x-5}$

$10^x = 10^{5x-10}$

$x = 5x - 10$

$\boxed{x = 2{,}5}$

2 rovnice, které je možné řešit dvěma postupy (1 příklad = ukázka)

$$9^{v+2}+5\cdot 9^{v+1}=14$$
$$9^v\cdot 81+5\cdot 9^v\cdot 9=14$$
$$9^v\cdot 81+45\cdot 9^v=14$$

Substituce: $9^v=x$

$$81x+45x=14$$
$$126x=14$$
$$x=\frac{1}{9}$$

$$9^v=\frac{1}{9}$$
$$9^v=9^{-1} \rightarrow \boxed{v=-1}$$

Jiné řešení:

$$9^v(81+45)=14$$
$$9^v\cdot 126=14$$
$$9^v=\frac{1}{9}$$
$$9^v=9^{-1}$$
$$\boxed{v=-1}$$

$$81^{-x}=27$$

$$(3^4)^{-x}=3^3$$
$$3^{-4x}=3^3$$
$$-4x=3$$
$$\boxed{x=-\frac{3}{4}}$$

Zkouška:

$L=81^{-(-\frac{3}{4})}=81^{\frac{3}{4}}=27$

$P=27$

$L=P$

$$3^{v-1}+3^{v-2}+3^{v-3}=13$$
$$3^v\cdot 3^{-1}+3^v\cdot 3^{-2}+3^v\cdot 3^{-3}=13$$
$$\frac{3^v}{3}+\frac{3^v}{9}+\frac{3^v}{27}=13$$

Sub. $3^v=x$

$$\frac{x}{3}+\frac{x}{9}+\frac{x}{27}=13 \quad |\cdot 27$$
$$9x+3x+x=351$$
$$13x=351$$
$$x=27$$

$$3^v=27$$
$$3^v=3^3$$
$$\boxed{v=3}$$

$$9^x-4\cdot 3^{x+1}=-27$$

$$(3^2)^x-4\cdot 3^x\cdot 3=-27$$
$$(3^x)^2-12\cdot 3^x=-27 \quad |\text{ sub. } 3^x=y$$

$$y^2-12y+27=0$$
$$y_{1,2}=\frac{12\pm\sqrt{144-108}}{2}$$
$$y_{1,2}=\frac{12\pm 6}{2}= \begin{cases} y_1=9 \\ y_2=3 \end{cases}$$

$3^x=3 \qquad 3^x=9$

$3^x=3^1 \qquad 3^x=3^2$

$\boxed{x=1} \qquad \boxed{x=2}$

$$\frac{6^{x^2}}{2^{-15}}=\frac{3^{-15}}{6^{12-12x}}$$

$$(2\cdot 3)^{x^2}\cdot 2^{15}=3^{-15}\cdot(2\cdot 3)^{-(12-12x)}$$
$$2^{x^2}\cdot 3^{x^2}\cdot 2^{15}=3^{-15}\cdot(2\cdot 3)^{12x-12}$$
$$2^{x^2}\cdot 3^{x^2}\cdot 2^{15}=3^{-15}\cdot 2^{12x-12}\cdot 3^{12x-12}$$
$$\frac{2^{x^2+15}}{2^{12x-12}}\cdot\frac{3^{x^2}}{3^{12x-27}}=1$$
$$2^{x^2+15}\cdot 2^{-12x+12}\cdot 3^{x^2}\cdot 3^{-12x+27}=1$$
$$2^{x^2-12x+27}\cdot 3^{x^2-12x+27}=1$$
$$6^{x^2-12x+27}=6^0$$
$$x^2-12x+27=0$$
$$x_{1,2}=\frac{12\pm\sqrt{36}}{2}$$
$$x_{1,2}=\frac{12\pm 6}{2}=\begin{cases} \boxed{x_1=9} \\ \boxed{x_2=3} \end{cases}$$

Zkouška pro $x_1=9$

$L=\frac{6^{81}}{2^{-15}}\doteq 3{,}5$

$P=\frac{3^{-15}}{6^{12-108}}=\frac{3^{-15}}{6^{-96}}\doteq 3{,}5$

$L=P$

Zkouška pro $x_2=3$

$L=\frac{6^{9}}{2^{-15}}\doteq 33$

$P=\frac{3^{-15}}{6^{12-36}}=\frac{3^{-15}}{6^{-24}}\doteq 3{,}3$

$L=P$

$$\frac{1}{5^{2x-4}} = 125$$

$$5^{-(2x-4)} = 5^3$$

$$-2x+4 = 3$$

$$-2x = -1$$

$$\boxed{x = 0{,}5}$$

$$25^x = \left(\frac{1}{5}\right)^{x^2}$$

$$(5^2)^x = \frac{1^{x^2}}{5^{x^2}}$$

$$5^{2x} = 5^{-(x^2)}$$

$$2x = -(x^2) \mid \cdot(-1)$$

$$-2x = x^2$$

$$x^2 + 2x = 0$$

$$x(x+2) = 0$$

$$\boxed{x_1 = 0} \lor x+2=0$$

$$\boxed{x_2 = -2}$$

$$\sqrt[3]{2^{2x-3}} = \sqrt[7]{0{,}5^{3-x}}$$

$$2^{\frac{2x-3}{3}} = \left(\frac{1}{2}\right)^{\frac{3-x}{7}}$$

$$2^{\frac{2x-3}{3}} = 2^{-\frac{3-x}{7}}$$

$$\frac{2x-3}{3} = \frac{-3+x}{7}$$

$$14x - 21 = -9 + 3x$$

$$11x = 12$$

$$\boxed{x = \frac{12}{11}}$$

$$4^x + 2^x - 6 = 0$$

$$(2^2)^x + 2^x - 6 = 0$$

$$(2^x)^2 + 2^x - 6 = 0$$

Sub. $2^x = y$

$$y^2 + y - 6 = 0$$

$$y_{1,2} = \frac{-1 \pm \sqrt{25}}{2}$$

$$y_{1,2} = \frac{-1 \pm 5}{2} \quad y_1 = 2, \quad y_2 = -3$$

$$2^x = 2 \qquad 2^x = -3$$

$$2^x = 2^1 \qquad \log 2^x = \log(-3)$$

$\boxed{x = 1}$ log. zápor. čísel neexistují

$$4^{(x+3)\cdot(2-5x)} = 1$$

$$4^{(x+3)\cdot(2-5x)} = 4^0$$

$(x+3)\cdot(2-5x) = 0$, platí:

$$x+3 = 0 \lor 2-5x = 0$$

$$\boxed{x_1 = -3} \qquad \boxed{x_2 = 0{,}4}$$

úloha má 2 řešení.

$$3^x = 7$$

$$\log 3^x = \log 7$$

$$x \cdot \log 3 = \log 7$$

$$\boxed{x = \frac{\log 7}{\log 3}} \doteq 1{,}7712\ldots$$

$$2^{4x} - 50 \cdot 2^{2x} = 896$$

$$2^{2\cdot 2x} - 50 \cdot 2^{2x} = 896$$

$$(2^{2x})^2 - 50 \cdot 2^{2x} = 896$$

Sub: $2^{2x} = y$

$$y^2 - 50 \cdot y - 896 = 0$$

$$y_{1,2} = \frac{50 \pm \sqrt{6084}}{2}$$

$$y_{1,2} = \frac{50 \pm 78}{2} \quad y_1 = 64, \quad y_2 = -14$$

$$2^{2x} = 64 \qquad 2^{2x} = -14$$

$$2^{2x} = 2^6 \qquad \log 2^{2x} = \log -14 \text{ neexistuje}$$

$$2x = 6$$

$\boxed{x = 3}$ Pouze 1 řešení.

$$2^x = \frac{4}{3}$$

NÁVOD:

Protože $\frac{4}{3}$ nelze napsat jako mocninu čísla 2 přímo, tak vyřešíme tuto rovnici logaritmováním:

$$\log 2^x = \log \frac{4}{3}$$

$$x \cdot \log 2 = \log 4 - \log 3$$

$$\boxed{x = \frac{\log 4 - \log 3}{\log 2}} \doteq 0{,}415\ldots$$

Zkouškou na kalkulačce lze potvrdit správnost řešení.

$$\sqrt[5]{\sqrt[x-3]{2}} = \sqrt{\sqrt[-x]{\frac{1}{2}}}$$

$$\left(2^{\frac{1}{x-3}}\right)^{\frac{1}{5}} = \left[\left(\frac{1}{2}\right)^{\frac{1}{-x}}\right]^{\frac{1}{2}}$$

$$2^{\frac{1}{5x-15}} = \left(\frac{1}{2}\right)^{-\frac{1}{2x}}$$

$$2^{\frac{1}{5x-15}} = 2^{\frac{1}{2x}}$$

$$\frac{1}{5x-15} = \frac{1}{2x}$$

$$5x-15 = 2x$$

$$3x = 15$$

$$\boxed{x=5}$$

$$5^{t-2} = \frac{10}{3}$$

$$5^t \cdot 5^{-2} = \frac{10}{3}$$

$$5^t \cdot \frac{1}{25} = \frac{10}{3}$$

$$5^t = \frac{250}{3}$$

$$\log 5^t = \log \frac{250}{3}$$

$$\log 5^t = \log 250 - \log 3$$

$$t \cdot \log 5 = \log 250 - \log 3$$

$$\boxed{t = \frac{\log 250 - \log 3}{\log 5}}$$

$$t \doteq 2{,}748\ldots$$

$$v^{\log v} = 100v$$

$$\log\left(v^{\log v}\right) = \log(100 \cdot v)$$

$$\log v \cdot \log v = \underbrace{\log 100}_{2} + \log v$$

$$\log^2 v = 2 + \log v$$

$$\log^2 v - \log v - 2 = 0$$

Sub. $\log v = x$

$$x^2 - x - 2 = 0$$

$$x_{1,2} = \frac{1 \pm \sqrt{9}}{2} = \frac{1 \pm 3}{2} \quad x_1 = 2,\; x_2 = -1$$

$\log v = 2$ | $\log_{10} v = -1$

$\log_{10} v = 2$ | $v = 10^{-1}$

$v = 10^2$ | $v = \frac{1}{10}$

$\boxed{v = 100}$ | $\boxed{v = 0{,}1}$

$$\left(\frac{9}{25}\right)^x \cdot \left(\frac{125}{27}\right)^{x-1} = \frac{\log 8}{\log 32}$$

$$\left(\frac{3^2}{5^2}\right)^x \cdot \left(\frac{5^3}{3^3}\right)^{x-1} = 0{,}6$$

$$\left(\frac{3}{5}\right)^{2x} \cdot \left(\frac{5}{3}\right)^{3(x-1)} = 0{,}6$$

$$\left(\frac{3}{5}\right)^{2x} \cdot \left(\frac{3}{5}\right)^{-3x+3} = 0{,}6$$

$$(0{,}6)^{-x+3} = 0{,}6^1$$

$$-x+3 = 1$$

$$\boxed{x=2}$$