Exponenciální rovnice řešené v r. 2008 ze Sbírky Ivana Bušo a ze zkoušek na VŠ (VŠE.)

Př. 9.1 185

$$3\cdot(4^x+9^{x+1}) = 2\left(3\cdot 4^{x+1} - \frac{9^{x+1}}{4}\right)$$

$$3\cdot(4^x+9^x\cdot 9^1) = 6\cdot 4^{x+1} - \frac{1}{2}\cdot 9^{x+1}$$

$$3\cdot 4^x + 27\cdot 9^x = 6\cdot 4^x\cdot 4^1 - \frac{1}{2}\cdot 9^x\cdot 9$$

$$3\cdot 4^x + 27\cdot 9^x = 24\cdot 4^x - \frac{9}{2}\cdot 9^x$$

$$27\cdot 9^x + \frac{9}{2}\cdot 9^x = 24\cdot 4^x - 3\cdot 4^x$$

$$9^x\left(27+\frac{9}{2}\right) = 4^x(24-3)$$

$$\frac{63}{2}\cdot 9^x = 21\cdot 4^x$$

$$\frac{9^x}{4^x} = 21 : \frac{63}{2}$$

$$\left(\frac{9}{4}\right)^x = \left(\frac{2}{3}\right)^1$$

$$\left(\frac{3^2}{2^2}\right)^x = \left(\frac{3}{2}\right)^{-1}$$

$$\left(\frac{3}{2}\right)^{2x} = \left(\frac{3}{2}\right)^{-1}$$

$$2x = -1$$

$$\boxed{x = -\frac{1}{2}}$$

Zkouška:

$$L = 3\cdot\left(4^{-\frac{1}{2}} + 9^{-\frac{1}{2}+1}\right) = 3\cdot\left[\left(\frac{1}{4}\right)^{\frac{1}{2}} + 9^{\frac{1}{2}}\right] =$$

$$= 3\cdot\left(\sqrt{\frac{1}{4}} + \sqrt{9}\right) = 3\cdot\left(\frac{1}{2}+3\right) = 3\cdot 3\frac{1}{2} = 10\frac{1}{2}$$

$$P = 2\cdot\left(3\cdot 4^{-\frac{1}{2}+1} - \frac{9^{-\frac{1}{2}+1}}{4}\right) =$$

$$= 2\cdot\left(3\cdot 4^{\frac{1}{2}} - \frac{1}{4}\cdot 9^{\frac{1}{2}}\right) = 2\cdot\left(3\sqrt{4} - \frac{1}{4}\sqrt{9}\right) =$$

$$= 2\cdot\left(3\cdot 2 - \frac{1}{4}\cdot 3\right) = 2\cdot\left(6 - \frac{3}{4}\right) =$$

$$= 2\cdot 5\frac{1}{4} = 10\frac{1}{2}$$

$L = P$ Řešením rovnice je

$$x = -\frac{1}{2}$$

Př. 9.2 186 Řešte rovnici:

$$2^x\cdot\left(\frac{1}{8}\right)^{1-x} + 2^{1-x}\cdot\left(\frac{1}{8}\right)^x = 1$$

$$2^x\cdot\left(\frac{1}{2^3}\right)^{1-x} + 2^{1-x}\cdot\left(\frac{1}{2^3}\right)^x = 1$$

$$\underbrace{2^x\cdot(2^{-3})^{1-x} + 2^{1-x}\cdot 2^{-3x} = 1}$$

$$2^x\cdot 2^{-3+3x} + 2^{1-x}\cdot 2^{-3x} = 1$$

$$2^{4x-3} + 2^{1-4x} = 1$$

$$\frac{2^{4x}}{2^3} + \frac{2}{2^{4x}} = 1 \quad \Big|\cdot 2^3\cdot 2^{4x}$$

$$2^{4x}\cdot 2^{4x} + 2\cdot 2^3 = 2^3\cdot 2^{4x}$$

$$2^{8x} + 2^4 = 2^3\cdot 2^{4x}$$

$$2^{8x} + 16 = 8\cdot 2^{4x}$$

$$2^{8x} - 8\cdot 2^{4x} + 16 = 0 \qquad \boxed{1}$$

Substituce: $2^{4x} = y \ldots 2^{8x} = 2^{4x}\cdot 2^{4x} = y\cdot y = y^2$

$$y^2 - 8y + 16 = 0$$

$$y_{1,2} = \frac{8 \pm \sqrt{64-64}}{2}$$

$y_1 = y_2 = 4$ dosadíme zpět:

$$2^{4x} = 4$$

$$2^{4x} = 2^2$$

$$4x = 2$$

$$\boxed{x = \frac{1}{2}}$$

Zkouška: $L = 2^{\frac{1}{2}} \cdot \left(\frac{1}{8}\right)^{1-\frac{1}{2}} + 2^{1-\frac{1}{2}} \cdot \left(\frac{1}{8}\right)^{\frac{1}{2}} = 2^{\frac{1}{2}} \cdot \left(\frac{1}{8}\right)^{\frac{1}{2}} + 2^{\frac{1}{2}} \cdot \left(\frac{1}{8}\right)^{\frac{1}{2}} =$
$2^{\frac{1}{2}} \cdot \left(\frac{1}{2^3}\right)^{\frac{1}{2}} + 2^{\frac{1}{2}} \cdot \left(\frac{1}{2^3}\right)^{\frac{1}{2}} = 2^{\frac{1}{2}} \cdot 2^{-\frac{3}{2}} + 2^{\frac{1}{2}} \cdot 2^{-\frac{3}{2}} = 2^{-1} + 2^{-1} = \frac{1}{2} + \frac{1}{2} = 1$

$P = 1$ ... $L = P$ Rovnice má řešení $x = \frac{1}{2}$.

9.3/187

$6 \cdot 6^x + \frac{6}{6^x} = 13 \quad | \cdot 6^x$

$6 \cdot 6^x 6^x + 6 = 13 \cdot 6^x$

Sub $6^x = y$

$6y \cdot y + 6 = 13y$

$6y^2 - 13y + 6 = 0$

$y_{1,2} = \frac{13 \pm \sqrt{169-144}}{12}$

$y_{1,2} = \frac{13 \pm 5}{12} = \begin{cases} \frac{3}{2} \\ \frac{2}{3} \end{cases}$

$6^x = \frac{3}{2} \quad \wedge \quad 6^x = \frac{2}{3}$

Obě rovnice zlogaritmujeme, neboť zlomky $\frac{3}{2}$ a $\frac{2}{3}$ nelze napsat jako mocniny čísla 6.

$x \cdot \log 6 = \log 3 - \log 2$

$\boxed{x_1 = \frac{\log 3 - \log 2}{\log 6}}$

$x \cdot \log 6 = \log 2 - \log 3$

$\boxed{x_2 = \frac{\log 2 - \log 3}{\log 6}}$

Zkouška je ve složce ve sl. 88, vynechávám ji

*9.4/189 $x^x - x^{-x} = 3(1 + x^{-x})$

$x^x - \frac{1}{x^x} = 3 + \frac{3}{x^x} \quad | \cdot x^x$

$x^x \cdot x^x - 1 = 3x^x + 3$

$x^x \cdot x^x - 3x^x - 4 = 0$ Sub. $x^x = y$

$y \cdot y - 3y - 4 = 0$

$y^2 - 3y - 4 = 0$

$y_{1,2} = \frac{3 \pm \sqrt{9+16}}{2} = \frac{3 \pm 5}{2} = \begin{cases} y_1 = 4 \\ y_2 = -1 \end{cases}$

Zpět

$x^x = 4 \quad \vee \quad x^x = -1$

$x^x = 2^2 \quad \vee \quad x^x = (-1)^{-1}$

$\boxed{x_1 = 2} \quad \vee \quad \boxed{x_2 = -1}$

Zkouška pro $x_1 = 2$

$L = 2^2 - 2^{-2} = 4 - \frac{1}{2^2} = 4 - \frac{1}{4} = 3\frac{3}{4}$

$P = 3 \cdot (1 + 2^{-2}) = 3 \cdot (1 + \frac{1}{4}) = 3 + \frac{3}{4} = 3\frac{3}{4}$

$L = P$

Zkouška pro $x_2 = -1$

$L = (-1)^{-1} - (-1)^1 = -\frac{1}{1^1} + 1 = -1 + 1 = 0$

$P = 3 \cdot [1 + (-1)^1] = 3 \cdot (1 - 1) = 3 \cdot 0 = 0$

$L = P$

9.7 a/93

$$3^x+3^{x+1}+3^{x+2}+3^{x+3}=\frac{40}{3}$$

$$3^x+3^x\cdot 3+3^x\cdot 3^2+3^x\cdot 3^3=\frac{40}{3}$$

$$3^x+3\cdot 3^x+9\cdot 3^x+27\cdot 3^x=\frac{40}{3}$$

$$3^x(1+3+9+27)=\frac{40}{3}$$

$$40\cdot 3^x=\frac{40}{3}\ \Big|\cdot\frac{1}{40}$$

$$3^x=\frac{1}{3}$$

$$3^x=3^{-1}$$

$$\boxed{x=-1}$$

b)

$$\underbrace{3^x+3^{x+1}+3^{x+2}+3^{x+3}}_{\text{viz vlevo}}=40$$

$$40\cdot 3^x=40$$

$$3^x=1$$

$$3^x=3^0$$

$$\boxed{x=0}$$

c)

$$3^x+3^{x+1}+3^{x+2}+3^{x+3}=80$$

$$40\cdot 3^x=80$$

$$3^x=2$$

$$\cancel{3}\cdot\log 3^x=\log 2$$

$$x\cdot\log 3=\log 2$$

$$\boxed{x=\frac{\log 2}{\log 3}}$$

$$x\doteq 0{,}630\,929\,753$$

9.8/93 Řešte v R rovnici

$$5\cdot 2^{x+2}-6\cdot 3^{x+2}=3^{x+3}+2\cdot 2^{x+1}$$

$$5\cdot 2^x\cdot 2^2-6\cdot 3^x\cdot 3^2=3^x\cdot 3^3+2\cdot 2^x\cdot 2^1$$

$$20\cdot 2^x-54\cdot 3^x=27\cdot 3^x+4\cdot 2^x$$

$$20\cdot 2^x-4\cdot 2^x=27\cdot 3^x+54\cdot 3^x$$

$$16\cdot 2^x=81\cdot 3^x$$

$$\frac{2^x}{3^x}=\frac{81}{16}$$

$$\left(\frac{2}{3}\right)^x=\frac{3^4}{2^4}$$

$$\left(\frac{2}{3}\right)^x=\left(\frac{3}{2}\right)^4$$

$$\left(\frac{2}{3}\right)^x=\left(\frac{2}{3}\right)^{-4}$$

$$\boxed{x=-4}$$

Zkouška:

$$L=5\cdot 2^{-4+2}-6\cdot 3^{-4+2}=5\cdot 2^{-2}-6\cdot 3^{-2}=$$

$$=5\cdot\frac{1}{2^2}-6\cdot\frac{1}{3^2}=\frac{5}{4}-\frac{6}{9}=\frac{5}{4}-\frac{2}{3}=\frac{7}{12}$$

$$P=3^{-4+3}+2\cdot 2^{-4+1}=3^{-1}+2\cdot 2^{-3}=\frac{1}{3}+2\cdot\frac{1}{8}=$$

$$=\frac{1}{3}+\frac{1}{4}=\frac{7}{12}\ ;\quad L=P$$

Př. 9.9/93

$$\sqrt[2x+7]{4^{13-x}}=1024$$

$$4^{\frac{13-x}{2x+7}}=4^5$$

$$\frac{13-x}{2x+7}=5$$

$$13-x=5\cdot(2x+7)$$

$$13-x=10x+35$$

$$11x=-22$$

$$\boxed{x=-2}$$

Pá. 9.10/93 Řešte v R rovnici

$$\sqrt[x]{81} + \frac{27}{\sqrt[x]{81}} = 12$$

$$\sqrt[x]{3^4} + \frac{3^3}{\sqrt[x]{3^4}} = 12$$

$$3^{\frac{4}{x}} + \frac{3^3}{3^{\frac{4}{x}}} = 12 \quad / \cdot 3^{\frac{4}{x}}$$

$$3^{\frac{4}{x}} \cdot 3^{\frac{4}{x}} + 27 = 12 \cdot 3^{\frac{4}{x}}$$

I. subst. $\frac{4}{x} = y$

$$3^y \cdot 3^y - 12 \cdot 3^y + 27 = 0$$

II. subst. $3^y = z$

$$z \cdot z - 12z + 27 = 0$$

$$z^2 - 12z + 27 = 0$$

$$z_{1,2} = \frac{12 \pm \sqrt{36}}{2} = \frac{12 \pm 6}{2} = \left\langle \begin{matrix} 9 \\ 3 \end{matrix} \right.$$

Zpět do II.

$$3^y = 9 \quad \vee \quad 3^y = 3$$

$$3^y = 3^2 \qquad 3^y = 3^1$$

$$y = 2 \qquad y = 1$$

Zpět do I

$$\frac{4}{x} = 2 \qquad \frac{4}{x} = 1$$

$$2x = 4 \qquad \boxed{x = 4}$$

$\boxed{x = 2}$ Dvě řešení.

Zkouška: pro $x=2$

$$L = \sqrt[2]{81} + \frac{27}{\sqrt[2]{81}} = 9 + \frac{27}{9} = 9 + 3 = 12$$

$P = 12 \ldots L = P$

Zkouška pro $x = 4$

$$L = \sqrt[4]{81} + \frac{27}{\sqrt[4]{81}} = 3 + \frac{27}{3} = 3 + 9 = 12$$

$P = 12 \ldots L = P$

9.11/93 Řešte v R rovnici:

$$\frac{2^x \cdot 3^{x+3}}{6^{7-x} \cdot 8^{x-4}} = 9^{x-2}$$

$$2^x \cdot 3^{x+3} \cdot 6^{x-7} \cdot 8^{4-x} = 9^{x-2}$$

$$2^x \cdot 3^x \cdot 3^3 \cdot 6^x \cdot 6^{-7} \cdot 8^4 \cdot 8^{-x} = 9^x \cdot 9^{-2}$$

$$2^x \cdot 3^x \cdot 3^3 \cdot (2 \cdot 3)^x \cdot 6^{-7} \cdot 8^4 \cdot \frac{1}{8^x} = 9^x \cdot \frac{1}{9^2}$$

$$\frac{2^x \cdot 3^x \cdot 3^3 \cdot 2^x \cdot 3^x \cdot 6^{-7} \cdot 8^4}{(2^3)^x} = (3^2)^x \cdot \frac{1}{(3^2)^2}$$

$$\frac{2^x \cdot 3^x \cdot 3^3 \cdot 2^x \cdot 3^x \cdot 6^{-7} \cdot 8^4}{2^{3x}} = 3^{2x} \cdot \frac{1}{3^4}$$

$$\frac{\cancel{2^{2x}} \cdot 3^{2x} \cdot 3^3 \cdot 6^{-7} \cdot 8^4}{\cancel{2^{2x}} \cdot 2^x} = 3^{2x} \cdot \frac{1}{3^4} \quad / \cdot \frac{1}{3^{2x}}$$

$$\frac{3^3 \cdot 6^{-7} \cdot 8^4}{2^x} = \frac{1}{3^4}$$

$$2^x = 3^3 \cdot 3^4 \cdot 6^{-7} \cdot 8^4$$

$$2^x = \frac{3^7 \cdot 8^4}{6^7}$$

$$2^x = 32$$

$$2^x = 2^5 \Rightarrow \boxed{x = 5}$$

Zkouška: $L = \frac{2^5 \cdot 3^8}{6^2 \cdot 8} = 729$

$P = 9^3 = 729$; $L = P$

Př. 9.12/93 Řešte v R rovnici:

$$81^x - 9^{x+1} = 3\boxed{\log_3 \frac{1}{27}} + 3^{2x}$$

Pomocný výpočet:

$$\log_3 \frac{1}{27} \Rightarrow 3^y = \frac{1}{27}$$

$$3^y = 3^{-3} \Rightarrow y = -3$$

$$81^x - 9^{x+1} = 3.(-3) + 3^{2x}$$

$$3^{4x} - 9^x.9 = -9 + 3^{2x}$$

$$(3^{2x})^2 - 3^{2x}.9 = -9 + 3^{2x}$$

Substituce: $3^{2x} = z$

$$z^2 - 9z = -9 + z$$

$$z^2 - 10z + 9 = 0$$

$$z_{1,2} = \frac{10 \pm \sqrt{64}}{2} = \frac{10 \pm 8}{2} \quad z_1 = 9, \; z_2 = 1$$

Zpět: $3^{2x} = 9 \;\vee\; 3^{2x} = 1$

$$3^{2x} = 3^2 \;\vee\; 3^{2x} = 3^0$$

$$2x = 2 \;\vee\; 2x = 0$$

$$\boxed{x = 1} \;\vee\; \boxed{x = 0}$$

Zkouška pro $x=1$

$$\left.\begin{array}{l} L = 81^1 - 9^2 = 81 - 81 = 0 \\ P = 3.(-3) + 3^2 = 9 - 9 = 0 \end{array}\right\} L = P$$

Zkouška pro $x=0$

$$\left.\begin{array}{l} L = 81^0 - 9^1 = 1 - 9 = -8 \\ P = 3.(-3) + 3^0 = -9 + 1 = -8 \end{array}\right\} L = P$$

$$2^{x+1} + 2^x - 3 = 0$$

$$2^x.2^1 + 2^x - 3 = 0$$

$$2^x(2+1) - 3 = 0$$

$$2^x.3 - 3 = 0$$

$$2^x.3 = 3$$

$$2^x = \frac{3}{3}$$

$$2^x = 1$$

$$2^x = 2^0$$

$$x = 0$$

Zkouška:

$$L = 2^{0+1} + 2^0 - 3 = 2 + 1 - 3 = 0$$

$$P = 0 \; \dots \; L = P$$

9.14/93 Řešte v R rovnici:

$$\sqrt{5^{3x} + 19} - \sqrt{5^{3x} - 4} = 1$$

Sub. $5^{3x} = y$

$$\sqrt{y + 19} - \sqrt{y - 4} = 1$$

$$\left(\sqrt{y + 19} - \sqrt{y - 4}\right)^2 = 1^2$$

$$y + 19 - 2\sqrt{y+19}.\sqrt{y-4} + y - 4 = 1$$

$$2y + 14 - 2.\sqrt{(y+19).(y-4)} = 0$$

$$-2\sqrt{(y+19).(y-4)} = -14 - 2y \quad | :(-2)$$

$$\left(\sqrt{(y+19).(y-4)}\right)^2 = (y+7)^2$$

$$\cancel{y^2} + 15y - 76 = \cancel{y^2} + 14y + 49$$

$$y = 125$$

Zpět: $5^{3x} = 125$

$$5^{3x} = 5^3$$

$$3x = 3$$

$$\boxed{x = 1}$$

Zkouška:

$$L = \sqrt{5^3 + 19} - \sqrt{5^3 - 4} = \sqrt{144} - \sqrt{121} =$$

$$12 - 11 = 1; \; P = 1; \; L = P$$

Následující příklady jsou ze zkoušek na vysoké školy:

$$\left(\frac{1}{4}\right)^{x-1}=16$$
$$\frac{\left(\frac{1}{4}\right)^{x}}{\left(\frac{1}{4}\right)^{1}}=16$$
$$\frac{\frac{1}{4^x}}{\frac{1}{4}}=16$$
$$\frac{4}{4^x}=16$$
$$4=16\cdot 4^x$$
$$4^x=\frac{4}{16}$$
$$4^x=\frac{1}{4}$$
$$4^x=4^{-1}$$
$$\boxed{x=-1}$$

Zkouška:

$$L=\left(\frac{1}{4}\right)^{-1-1}=\left(\frac{1}{4}\right)^{-2}=4^2=16$$
$$P=16;\ L=P$$

$$3^{x+2}-3^x=8$$
$$3^x\cdot 3^2-3^x=8$$
$$3^x(3^2-1)=8$$
$$3^x\cdot 8=8$$
$$3^x=\frac{8}{8}$$
$$3^x=1$$
$$3^x=3^0$$
$$\boxed{x=0}$$

Zkouška:

$$L=3^{0+2}-3^0=3^2-1=8$$
$$P=8;\ L=P$$

$$4^{x-4}-11\cdot 4^{x-6}=5$$
$$\frac{4^x}{4^4}-11\cdot\frac{4^x}{4^6}=5$$
$$4^x\left(\frac{1}{256}-\frac{11}{4096}\right)=5$$
$$4^x\cdot\frac{16-11}{4096}=5$$
$$4^x\cdot\frac{5}{4096}=5$$
$$4^x\cdot 5=20480$$
$$4^x=4096$$
$$4^x=4^6$$
$$\boxed{x=6}$$

Zkouška: $L=4^{6-4}-11\cdot 4^{6-6}=4^2-11=16-11=5,\ P=5$

$L=P$

$$4^{x+5}-9\cdot 4^{x+3}=7$$
$$4^x\cdot 4^5-9\cdot 4^x\cdot 4^3=7$$
$$4^x(1024-9\cdot 64)=7$$
$$4^x\cdot 448=7$$
$$4^x=\frac{7}{448}$$
$$4^x=\frac{1}{64}$$
$$4^x=\frac{1}{4^3}$$
$$4^x=4^{-3}$$
$$\boxed{x=-3}$$

Zkouška:

$$L=4^{-3+5}-9\cdot 4^0=4^2-9=7$$
$$P=7;\ L=P$$

⊗⊗ Př. ze sbírky pro vysoké školy, totéž platí o dalších příkladech

Řešte v R:

$$4^{x+\sqrt{x^2-2}}-5\cdot 2^{x+\sqrt{x^2-2}}\cdot 2^{-1}=6$$
$$4^x\cdot 4^{\sqrt{x^2-2}}-5\cdot 2^{x+\sqrt{x^2-2}}\cdot 2^{-1}=6$$
$$(2^2)^x\cdot(2^2)^{\sqrt{x^2-2}}-5\cdot 2^{x+\sqrt{x^2-2}}\cdot 2^{-1}=6$$
$$(2^x)^2\cdot 2^{2\cdot\sqrt{x^2-2}}-5\cdot 2^{x+\sqrt{x^2-2}}\cdot 2^{-1}=6$$
$$\left(2^x\cdot 2^{\sqrt{x^2-2}}\right)^2-5\cdot 2^{x+\sqrt{x^2-2}}\cdot 2^{-1}=6$$
$$\left(2^{x+\sqrt{x^2-2}}\right)^2-5\cdot 2^{x+\sqrt{x^2-2}}\cdot 2^{-1}=6$$

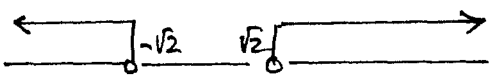
Podmínka:

$$x^2-2\geq 0$$
$$x^2\geq 2$$
$$|x|\geq\sqrt{2}$$

Pro $x>0$ je $|x|>\sqrt{2}\ \dots\ x>\sqrt{2}$

" $x<0$ je $|x|<\sqrt{2}\ \dots\ -x>\sqrt{2}$

$x<-\sqrt{\ }$



$$x\in(-\infty;-\sqrt{2})\cup(\sqrt{2};\infty)$$

Sub.: $2^{x+\sqrt{x^2-2}}=y$

$y^2-5y\cdot\frac{1}{2}=6$

$y^2-\frac{5}{2}y-6=0 \quad |\cdot 2$

$2y^2-5y-12=0$

$y_{1,2}=\frac{5\pm\sqrt{121}}{4}$

$y_{1,2}=\frac{5\pm 11}{4}$

$= \begin{cases} y_1=4 \\ y_2=-\frac{3}{2} \text{ (nevyh.)} \end{cases}$

Spät.: $2^{x+\sqrt{x^2-2}}=4$

$2^{x+\sqrt{x^2-2}}=2^2$

$x+\sqrt{x^2-2}=2$

$\sqrt{x^2-2}\overset{*}{=}2-x$

$\cancel{x^2}-2=4-4x+\cancel{x^2}$

$4x=6$

$\boxed{x=\frac{3}{2}}$

Skouška:

$L=4^{\frac{3}{2}+\sqrt{\frac{9}{4}-2}}-5\cdot 2^{\frac{3}{2}+\sqrt{\frac{9}{4}-2}}\cdot 2^{-1}=$

$=4^{\frac{3}{2}+\sqrt{\frac{1}{4}}}-5\cdot 2^{\frac{3}{2}+\sqrt{\frac{1}{4}}}\cdot\frac{1}{2}=$

$4^{\frac{3}{2}+\frac{1}{2}}-5\cdot 2^{\frac{3}{2}+\frac{1}{2}}\cdot\frac{1}{2}=$

$=4^2-5\cdot 2^2\cdot\frac{1}{2}=16-5\cdot 4\cdot\frac{1}{2}=$

$16-10=6$

$P=6;\ L=P$

$2^{3x}=16$

$2^{3x}=2^4$

$3x=4$

$\boxed{x=\frac{4}{3}}$

$\left(\frac{3}{5}\right)^x=\left(1\frac{2}{3}\right)^3$

$\left(\frac{3}{5}\right)^x=\left(\frac{5}{3}\right)^3$

$\left(\frac{3}{5}\right)^x=\left(\frac{3}{5}\right)^{-3}$

$\boxed{x=-3}$

$0{,}6^x+0{,}6^{x-1}=\frac{40}{9}$

$\underline{0{,}6^x}+\underline{0{,}6^x\cdot 0{,}6^{-1}}=\frac{40}{9}$

$0{,}6^x\cdot\left(1+\frac{1}{0{,}6}\right)=\frac{40}{9}$

$0{,}6^x\cdot\frac{8}{3}=\frac{40}{9}$

$0{,}6^x=\frac{40}{9}:\frac{8}{3}$

$0{,}6^x=\frac{5}{3}$

$\left(\frac{6}{10}\right)^x=\left(\frac{3}{5}\right)^{-1}$

$\left(\frac{3}{5}\right)^x=\left(\frac{3}{5}\right)^{-1}$

$\boxed{x=-1}$

Zkouška pro $x=-1$

$L=0{,}6^{-1}+0{,}6^{-1-1}=\left(\frac{6}{10}\right)^{-1}+\left(\frac{6}{10}\right)^{-2}=$

$=\left(\frac{10}{6}\right)^1+\left(\frac{10}{6}\right)^2=\frac{10}{6}+\frac{100}{36}=\frac{40}{9}$

$P=\frac{40}{9};\ L=P$

$3^{5x}=5^{3x}$

$(3^5)^x=(5^3)^x$

$243^x=125^x$

Rovnost nastane jen, když

$\boxed{x=0}$

$\left(1-\frac{5}{9}\right)^{\frac{2}{3-2x}}=\left(\frac{9}{4}\right)^{\frac{3}{x-5}}$

$\left(\frac{4}{9}\right)^{\frac{2}{3-2x}}=\left(\frac{4}{9}\right)^{-\frac{3}{x-5}}$

$\frac{2}{3-2x}=-\frac{3}{x-5}$

$2(x-5)=-3(3-2x)$

$2x-10=-9+6x$

$4x=-1$

$\boxed{x=-\frac{1}{4}}$

Zkouška pro $x=-\frac{1}{4}$

$L=\left(1-\frac{5}{9}\right)^{\frac{2}{3+\frac{1}{2}}}=$

$=\left(\frac{4}{9}\right)^{\frac{\frac{2}{1}}{\frac{7}{2}}}=\left(\frac{4}{9}\right)^{\frac{4}{7}}$

$P=\left(\frac{9}{4}\right)^{\frac{\frac{3}{1}}{\frac{21}{4}}}=\left(\frac{9}{4}\right)^{\frac{12}{21}}=\left(\frac{9}{4}\right)^{\frac{4}{7}}$

$L=P$

$5^{x-4} = 0{,}008$

$5^{x-4} = \frac{8}{1000}$

$5^{x-4} = \frac{8}{10^3}$

$5^{x-4} = \frac{8}{(5.2)^3}$

$5^{x-4} = \frac{8}{5^3.2^3}$

$5^{x-4} = 5^{-3}$

$x-4 = -3$

$\boxed{x=1}$

$\frac{2^{2x+2}}{2^{3x-5}} = \frac{\log 16}{\log 4}$ (na kalkul.)

$\frac{2^{2x+2}}{2^{3x-5}} = 2$

$2^{2x+2} = 2^1 . 2^{3x-5}$

$2^{2x+2} = 2^{1+3x-5}$

$2^{2x+2} = 2^{3x-4}$

$2x+2 = 3x-4$

$x = 6$

Skouška:

$L = \frac{2^{14}}{2^{13}} = 2$; $P = 2$; $L = P$

$\left(\frac{4}{25}\right)^{x+3} . \left(\frac{125}{8}\right)^{4x-1} = \frac{5}{2}$

$\left(\frac{2^2}{5^2}\right)^{x+3} . \left(\frac{5^3}{2^3}\right)^{4x-1} = \frac{5}{2}$

$\left[\left(\frac{2}{5}\right)^2\right]^{x+3} . \left[\left(\frac{5}{2}\right)^3\right]^{4x-1} = \frac{5}{2}$

$\left(\frac{2}{5}\right)^{2x+6} . \left(\frac{5}{2}\right)^{12x-3} = \frac{5}{2}$

$\left(\frac{5}{2}\right)^{-2x-6} . \left(\frac{5}{2}\right)^{12x-3} = \left(\frac{5}{2}\right)^1$

$\left(\frac{5}{2}\right)^{-2x-6+12x-3} = \left(\frac{5}{2}\right)^1$

$\left(\frac{5}{2}\right)^{10x-9} = \left(\frac{5}{2}\right)^1$

$10x-9 = 1$

$10x = 10$

$\boxed{x=1}$

Skouška:

$L = \left(\frac{4}{25}\right)^4 . \left(\frac{125}{8}\right)^3 = 0{,}16^4 . 15{,}625^3 = 2{,}5$

$P = \frac{5}{2} = 2{,}5$; $L = P$

$\sqrt[2x+7]{4^{13-x}} = 1024$

$4^{\frac{13-x}{2x+7}} = 4^5$

$\frac{13-x}{2x+7} = 5$

$13-x = 10x+35$

$11x = -22$

$\boxed{x=-2}$

Zkouška:

$L = \sqrt[3]{4^{15}} = 4^{\frac{15}{3}} = 4^5 = 1024$; $P = 1024$; $L = P$

⊛⊛ $\frac{2^{x+3} . 3^{x+2}}{6^{7-x} . 8^{x-1}} = \frac{9^{x-2}}{3}$

$\frac{2^{x+3} . 3^{x+2}}{(2.3)^{7-x} . (2^3)^{x-1}} = \frac{(3^2)^{x-2}}{3}$

Pokračuj na $\boxed{9}$.

$$\frac{2^{x+3}\cdot 3^{x+2}}{2^{7-x}\cdot 3^{7-x}\cdot 2^{3x-3}} = \frac{3^{2x-4}}{3}$$

$$\underbrace{2^{x+3}\cdot 2^{-7+x}\cdot 2^{-3x+3}}_{2^{-x-1}} \cdot \underbrace{3^{x+2}\cdot 3^{-7+x}}_{3^{2x-5}} = \underbrace{3^{2x-4}\cdot 3^{-1}}_{3^{2x-5}}$$

$$= 3^{2x-5}$$

$$2^{-x-1}\cdot 3^{2x-5} = 3^{2x-5}$$

$$2^{-x-1} = \frac{3^{2x-5}}{3^{2x-5}}$$

$$2^{-x-1} = 1$$

$$2^{-x-1} = 2^0$$

$$-x-1 = 0$$

$$-x = 1$$

$$\boxed{x=-1}$$

Zkouška:

$$L = \frac{2^2\cdot 3}{6^8\cdot 8^{-2}} = \frac{2^2\cdot 3}{(2\cdot 3)^8\cdot (2^3)^{-2}} = \frac{2^2\cdot 3}{2^8\cdot 3^8\cdot 2^{-6}} = \frac{2^2\cdot 3}{2^2\cdot 3^8} = \frac{3}{3^8} = \frac{1}{3^7}$$

$$P = \frac{3^{-6}}{3} = \frac{1}{3^6\cdot 3} = \frac{1}{3^7};\ L=P$$

$\circledast$ $\left(\frac{4}{9}\right)^x\cdot\left(\frac{27}{8}\right)^{x-1} = \frac{\log 4}{\log 8}$ (na kalkulačce)

$$\left[\left(\frac{2}{3}\right)^2\right]^x\cdot\left[\left(\frac{3}{2}\right)^3\right]^{x-1} = \frac{2}{3}$$

$$\left(\frac{2}{3}\right)^{2x}\cdot\left(\frac{3}{2}\right)^{3x-3} = \frac{2}{3}$$

$$\left(\frac{2}{3}\right)^{2x}\cdot\left(\frac{2}{3}\right)^{-3x+3} = \frac{2}{3}$$

$$\left(\frac{2}{3}\right)^{2x-3x+3} = \frac{2}{3}$$

$$\left(\frac{2}{3}\right)^{-x+3} = \left(\frac{2}{3}\right)^1$$

$$-x+3 = 1$$

$$\boxed{x=2}$$

Zkouška: $L=\left(\frac{4}{9}\right)^2\cdot\left(\frac{27}{8}\right)^1 = \frac{16}{81}\cdot\frac{27}{8} = \frac{2}{3}$; $P=\frac{2}{3}$

$L=P$

$\circledast$

$$\frac{10^{x^2}}{2^{-15}} = \frac{5^{-15}}{10^{12-12x}}$$

$$\frac{(2\cdot 5)^{x^2}}{2^{-15}} = \frac{5^{-15}}{(2\cdot 5)^{12-12x}}$$

$$\frac{2^{x^2}\cdot 5^{x^2}}{2^{-15}} = \frac{5^{-15}}{2^{12-12x}\cdot 5^{12-12x}}$$

$$2^{x^2}\cdot 5^{x^2}\cdot 2^{12-12x}\cdot 5^{12-12x} = 5^{-15}\cdot 2^{-15}$$

$$2^{x^2-12x+12}\cdot 5^{x^2-12x+12} = 10^{-15}$$

Substituce: $x^2-12x+12=y$

$$2^y \cdot 5^y = 10^{-15}$$
$$10^y = 10^{-15}$$
$$y = -15 \text{ zpět}$$

$$x^2 - 12x + 12 = -15$$
$$x^2 - 12x + 27 = 0$$
$$x_{1,2} = \frac{12 \pm \sqrt{36}}{2} = \frac{12 \pm 6}{2} = \boxed{x_1 = 9},\ \boxed{x_2 = 3}$$

Zkouška pro x_1: $L = \dfrac{10^{81}}{2^{-15}} = 2^{81} \cdot 5^{81} \cdot 2^{15} = 2^{96} \cdot 5^{81}$

$$P = \frac{5^{-15}}{10^{+12-108}} = \frac{5^{-15}}{10^{-96}} = \frac{5^{-15}}{2^{-96} \cdot 5^{-96}} = 5^{-15} \cdot 5^{96} \cdot 2^{96} =$$
$$= 2^{96} \cdot 5^{81};\ L = P$$

Zkouška pro x_2: $L = \dfrac{10^9}{2^{-15}} = 2^9 \cdot 5^9 \cdot 2^{15} = 5^9 \cdot 2^{24}$

$$P = \frac{5^{-15}}{10^{12-36}} = \frac{5^{-15}}{10^{-24}} = \frac{5^{-15}}{2^{-24} \cdot 5^{-24}} = 5^{-15} \cdot 5^{24} \cdot 2^{24} = 5^9 \cdot 2^{24}$$
$$L = P$$

Řešte rovnici:

$$2^{\frac{3}{\log_2 x}} = \frac{1}{64}$$
$$2^{\frac{3}{\log_2 x}} = 2^{-6}$$
$$\frac{3}{\log_2 x} = -6$$
$$\log_2 x = -\frac{1}{2}$$

Subst. $\log_2 x = y$

$y = -\frac{1}{2}$ zpět

$$\log_2 x = -\frac{1}{2} \Rightarrow$$

$$x = 2^{-\frac{1}{2}}$$
$$x = \frac{1}{2^{\frac{1}{2}}}$$
$$x = \frac{1}{\sqrt{2}}$$
$$x = \frac{\sqrt{2}}{2}$$

Zkouška:

$$L = 2^{\frac{3}{-\frac{1}{2}}} = 2^{-6}$$
$$P = \frac{1}{64} = 2^{-6}$$
$$L = P$$

Řešte rovnici: ⊛

$$\sqrt[x+\frac{1}{2}]{729} = \sqrt[x-\frac{1}{2}]{9}$$
$$\sqrt[x+\frac{1}{2}]{3^6} = \sqrt[x-\frac{1}{2}]{3^2}$$
$$3^{\frac{6}{x+\frac{1}{2}}} = 3^{\frac{x-\frac{1}{2}}{2}}$$
$$\frac{6}{x+\frac{1}{2}} = \frac{x-\frac{1}{2}}{2}$$
$$\left(x+\frac{1}{2}\right)\cdot\left(x-\frac{1}{2}\right) = 12$$
$$x^2 - \frac{1}{4} = 12$$
$$x^2 = \frac{49}{4}$$

$$x_{1,2} = \pm\frac{7}{2} = \boxed{x_1 = \frac{7}{2}}$$
$x_2 = -\frac{7}{2}$ není řešitelné v R

Ve výsledcích je chybně uveden výsledek $x = 1$ (viz zkouška).

$L = \sqrt[\frac{7}{2}+\frac{1}{2}]{729} = \sqrt[4]{729} = \sqrt[4]{3^6} = 3^{\frac{4}{6}} = 3^{\frac{2}{3}}$; $P = \sqrt[\frac{7}{2}-\frac{1}{2}]{9} = \sqrt[3]{3^2} = 3^{\frac{2}{3}}$; $L=P$

Řešte rovnici:

$27^{5x-6} \cdot 81^{2x+3} = 9^{4x-2} \cdot 3^{7x-2}$

$(3^3)^{5x-6} \cdot (3^4)^{2x+3} = (3^2)^{4x-2} \cdot 3^{7x-2}$

$3^{15x-18} \cdot 3^{8x+12} = 3^{8x-4} \cdot 3^{7x-2}$

$3^{23x-6} = 3^{15x-6}$

$23x-6 = 15x-6$

$13x = 0$

$\boxed{x=0}$

Zkouška:

$L = 27^{-6} \cdot 81^3 = \frac{81^3}{27^6} = \frac{(3^4)^3}{(3^3)^6} = \frac{3^{12}}{3^{18}} = \frac{1}{3^6} = 3^{-6}$

$P = 9^{-2} \cdot 3^{-2} = (3^2)^{-2} \cdot 3^{-2} = 3^{-4} \cdot 3^{-2} = 3^{-6}$

$L=P$

Řešte rovnici:

$\sqrt[x]{81} + \frac{27}{\sqrt[x]{81}} = 12$

Sub. $\sqrt[x]{81} = y$

$y + \frac{27}{y} = 12 \quad | \cdot y$

$y^2 + 27 = 12y$

$y^2 - 12y + 27 = 0$

$y_{1,2} = \frac{12 \pm \sqrt{144-108}}{2}$

$y_{1,2} = \frac{12 \pm 6}{2} = \begin{cases} y_1 = 9 \\ y_2 = 3 \end{cases}$

Zpět:

$\sqrt[x]{81} = 9$	$\sqrt[x]{81} = 3$
$\sqrt[x]{3^4} = 3^2$	$\sqrt[x]{3^4} = 3$
$3^{\frac{4}{x}} = 3^2$	$3^{\frac{4}{x}} = 3^1$
$\frac{4}{x} = 2$	$\frac{4}{x} = 1$
$\boxed{x_1 = 2}$	$\boxed{x_2 = 4}$

Zkouška: pro x_1:

$\sqrt{81} + \frac{27}{\sqrt{81}} = 9 + \frac{27}{9}$

$9+3 = 12$; $P=12$; $L=P$

Zkouška pro x_2: $L = \sqrt[4]{81} + \frac{27}{\sqrt[4]{81}} = 3 + \frac{27}{3}$

$= 3 + 9 = 12$; $L=P$

Řešte rovnici:

$\frac{2^x \cdot 3^{x+3}}{6^{7-x} \cdot 8^{x-4}} = 9^{x-2}$

$\frac{2^x \cdot 3^{x+3}}{(2 \cdot 3)^{7-x} \cdot (2^3)^{x-4}} = (3^2)^{x-2}$

$\frac{2^x \cdot 3^{x+3}}{2^{7-x} \cdot 3^{7-x} \cdot 2^{3x-12}} = 3^{2x-4}$

$2^x \cdot 2^{-7+x} \cdot 2^{-3x+12} \cdot 3^{x+3} \cdot 3^{-7+x} = 3^{2x-4}$

$2^{-x+5} \cdot 3^{2x-4} = 3^{2x-4}$

$2^{-x+5} = \frac{3^{2x-4}}{3^{2x-4}}$

$2^{-x+5} = 1$

$2^{-x+5} = 2^0$

$-x+5 = 0$

$\boxed{x=5}$

Zkouška: $L = \frac{2^5 \cdot 3^8}{6^2 \cdot 8} = \frac{2^5 \cdot 3^8}{2^2 \cdot 3^2 \cdot 2^3} = \frac{2^5 \cdot 3^8}{2^5 \cdot 3^2} = 3^6 = 729$; $P = 9^{5-2} = 9^3 = 729$; $L=P$

Řešte rovnici ⊗⊘

$$\sqrt{5^{3x}+19}-\sqrt{5^{3x}-4}=1$$

$$\sqrt{5^{3x}+19}=1-\sqrt{5^{3x}-4}$$

$$\left(\sqrt{5^{3x}+19}\right)^2=\left(1-\sqrt{5^{3x}-4}\right)^2$$

$$\cancel{5^{3x}}+19=1-2\sqrt{5^{3x}-4}+\cancel{5^{3x}}-4$$

$$22=-2\sqrt{5^{3x}-4}$$

$$2\sqrt{5^{3x}-4}=-22$$

$$\sqrt{5^{3x}-4}=-11 \quad \text{umocni}$$

$$5^{3x}-4=121$$

$$5^{3x}=125$$

$$5^{3x}=5^3$$

$$3x=3$$

$$\boxed{x=1}$$

Zkouška:

$L=\sqrt{5^3+19}-\sqrt{5^3-4}=\sqrt{144}-\sqrt{121}=12-11=1$

$P=1$

$L=P$

Řešte v R:

$$3^{2x-1}+3^x-3^0=3^{-1}$$

$$3^{2x}\cdot 3^{-1}+3^x-1=\frac{1}{3}$$

$$(3^x)^2\cdot\frac{1}{3}+3^x-\frac{4}{3}=0$$

Subst. $3^x=y$

$$y^2\cdot\frac{1}{3}+y-\frac{4}{3}=0 \quad |\cdot 3$$

$$y^2+3y-4=0$$

$$y_{1,2}=\frac{-3\pm\sqrt{9+16}}{2}=\frac{-3\pm\sqrt{25}}{2}=\frac{-3\pm 5}{2}= \begin{cases} y_1=1 \\ y_2=-4 \end{cases}$$

Zpět: $3^x=1$ $\qquad$ $3^x=-4$

$3^x=3^0$ $\qquad$ nemá řešení

$$\boxed{x=0}$$

Zkouška: $L=3^{-1}+\cancel{3^0}-\cancel{3^0}=3^{-1}$; $P=3^{-1}$; $L=P$

Řešte v R:

$$4^{x+1}-8\cdot 4^{x-1}=32$$

$$4^x\cdot 4-8\cdot 4^x\cdot 4^{-1}=32$$

Sub. $4^x=y$

$$4y-2y=32$$

$$2y=32$$

$$y=16$$

Zpět

$$4^x=16$$

$$4^x=4^2$$

$$\boxed{x=2}$$

Zkouška: $L=4^3-8\cdot 4=64-32=32$; $P=32$

$L=P$

Řešte v R:

$$5^x+1-3\cdot 5^x=-49$$

Substituce: $5^x=y$

$$y+1-3y+49=0$$

$$-2y+50=0 \quad |:(-2)$$

$$y=25$$

Zpět: $5^x=25$

$5^x=5^2$

$$\boxed{x=2}$$

Zkouška:

$L=5^2+1-3\cdot 5^2=25+1-75=-49$

$P=-49$; $L=P$

Řešte v R:

$$4\cdot 3^{x+1}-3^{x-1}=315$$

$$4\cdot 3^x\cdot 3-3^x\cdot 3^{-1}=315$$

$$12\cdot 3^x-\frac{1}{3}\cdot 3^x=315$$

Subst. $3^x=y$

$$12y-\frac{1}{3}y=315 \quad |\cdot 3$$

$$36y-y=945$$

$$35y=945$$

$$y=27$$

Zpět: $3^x=27$

$$3^x=3^3$$

$$\boxed{x=3}$$

Zkouška:

$L=4\cdot 3^4-3^2=315$

$P=315$; $L=P$

Řešte v R:

$$5\cdot 4^{x+1}-4^{x+2}=4^{x-1}+240$$

$$5\cdot 4^x\cdot 4-4^x\cdot 4^2=4^x\cdot 4^{-1}+240$$

$$20\cdot 4^x-16\cdot 4^x=\frac{1}{4}\cdot 4^x+240$$

Substituce: $4^x=y$

$$20y-16y=\frac{1}{4}y+240$$

$$4y-\frac{1}{4}y=240$$

$$\frac{15}{4}y=240$$

$$y=240\cdot\frac{4}{15}$$

$$y=64$$

Zpět:

$$4^x=64$$

$$4^x=4^3$$

$$\boxed{x=3}$$

Zkouška: $L=5\cdot 4^4-4^5=256$

$P=4^2+240=256$

$L=P$

⊛ $25^{2x}-3\cdot 25^x=10$

$$(5^2)^{2x}-3\cdot(5^2)^x=10$$

$$5^{4x}-3\cdot 5^{2x}=10$$

$$(5^x)^4-3(5^x)^2-10=0$$

1. substituce: $5^x=y$

$$y^4-3y^2-10=0$$

2. sub. $y^2=z$

$$z^2-3z-10=0$$

$$z_{1,2}=\frac{3\pm\sqrt{49}}{2}$$

$$z_{1,2}=\frac{3\pm 7}{2} \begin{cases} z_1=5 \\ z_2=-1 \text{ (ne)} \end{cases}$$

zpět do ② sub. $y^2=5$

$$y_{1,2}=\pm\sqrt{5}$$

zpět do ①. $5^x=5^{\frac{1}{2}}$

$$\boxed{x=\frac{1}{2}}$$

$5^x=-5^{\frac{1}{2}}$ nevyhovuje

Zkouška: $L=25^{2\cdot\frac{1}{2}}-3\cdot 25^{\frac{1}{2}}=25-3\cdot 5=25-15=10$; $P=10$; $L=P$

Řešte v R: ⊛

$$3^{2x-1}+3^{2x-2}-3^{2x-4}=315$$

$$3^{2x}\cdot 3^{-1}+3^{2x}\cdot 3^{-2}-3^{2x}\cdot 3^{-4}=315$$

Substituce: $3^{2x}=y$

$$\frac{1}{3}y+\frac{1}{9}y-\frac{1}{81}y=315 \quad |\cdot 81$$

$$27y+9y-y=25515$$

$$35y=25515$$

$y=729\ (=3^6)$ Dalších.

Opak.: $3^{2x} = 3^6$ Zkouška: $L = 3^5 + 3^4 - 3^2 = 243 + 81 - 9 = 315$

$2x = 6$ $P = 315;\ L = P$

$\boxed{x = 3}$

---

⑧ Řešte v R:

$5 \cdot 2^{x+2} - 6 \cdot 3^{x+2} = 3^{x+3} + 2 \cdot 2^{x+1}$

$5 \cdot 2^x \cdot 4 - 6 \cdot 3^x \cdot 9 = 3^x \cdot 27 + 2 \cdot 2^x \cdot 2$

$20 \cdot 2^x - 54 \cdot 3^x = 27 \cdot 3^x + 4 \cdot 2^x$

$16 \cdot 2^x = 81 \cdot 3^x$

$\frac{2^x}{3^x} = \frac{81}{16}$

$\left(\frac{2}{3}\right)^x = \left(\frac{3}{2}\right)^4$

$\left(\frac{2}{3}\right)^x = \left(\frac{2}{3}\right)^{-4}$

$\boxed{x = -4}$ Zkouška:

$L = 5 \cdot 2^{-2} - 6 \cdot 3^{-2} =$

$5 \cdot \frac{1}{4} - 6 \cdot \frac{1}{9} = \frac{5}{4} - \frac{2}{3} = \frac{7}{12}$

$P = 3^{-1} + 2 \cdot 2^{-3} = \frac{1}{3} + 2 \cdot \frac{1}{8} = \frac{7}{12}$

$L = P$

Řešte v R:

$4^x - 10 \cdot 2^{x-1} = 24$

$(2^2)^x - 10 \cdot 2^x \cdot 2^{-1} = 24$

$(2^x)^2 - 5 \cdot 2^x = 24$

Subst. $2^x = y$

$y^2 - 5y - 24 = 0$

$y_{1,2} = \frac{5 \pm \sqrt{25 + 96}}{2} = \frac{5 \pm 11}{2} = \begin{cases} y_1 = 8 \\ y_2 = -3 \end{cases}$

$2^x = 8$ ... $2^x = -3$ nemá řešení

$2^x = 2^3$ Zkouška:

$\boxed{x = 3}$ $L = 4^3 - 10 \cdot 2^2 = 24$; $P = 24$; $L = P$

⊗⊗ Řešte v R:

$11^{3x-2} + 13^{3x-2} = 13^{3x-1} - 11^{3x-1}$

$11^{3x} \cdot 11^{-2} + 13^{3x} \cdot 13^{-2} = 13^{3x} \cdot 13^{-1} - 11^{3x} \cdot 11^{-1}$

$\frac{11^{3x}}{121} + \frac{13^{3x}}{169} = \frac{13^{3x}}{13} - \frac{11^{3x}}{11}$

$\frac{11^{3x}}{121} + \frac{11^{3x}}{11} = \frac{13^{3x}}{13} - \frac{13^{3x}}{169}$

$\frac{11^{3x} + 11 \cdot 11^{3x}}{121} = \frac{13 \cdot 13^{3x} - 13^{3x}}{169}$

$\frac{11^{3x} \cdot (1 + 11)}{121} = \frac{13 \cdot 13^{3x} - 13^{3x}}{169}$

$\frac{12}{121} \cdot 11^{3x} = \frac{13^{3x} \cdot (13 - 1)}{169}$

$\frac{12 \cdot 11^{3x}}{121} = \frac{12}{169} \cdot 13^{3x}$

$\frac{11^{3x}}{13^{3x}} = \frac{12}{169} \cdot \frac{121}{12}$

$\left(\frac{11}{13}\right)^{3x} = \frac{121}{169}$

$\left(\frac{11}{13}\right)^{3x} = \left(\frac{11}{13}\right)^2 \Rightarrow 3x = 2 \Rightarrow \boxed{x = \frac{2}{3}}$

Zkouška: $L = 11^0 + 13^0 = 1 + 1 = 2$

$P = 13^1 - 11^1 = 13 - 11 = 2$; $L = P$

Řešte rovnici:

$$3\cdot 4^{-x}+\frac{1}{3}\cdot 9^{2-x}=6\cdot 4^{1-x}-\frac{1}{2}\cdot 9^{1-x}$$

$$3\cdot(2^2)^{-x}+\frac{1}{3}\cdot(3^2)^{2-x}=6\cdot(2^2)^{1-x}-\frac{1}{2}\cdot(3^2)^{1-x}$$

$$3\cdot 2^{-2x}+\frac{1}{3}\cdot 3^4\cdot 3^{-2x}=6\cdot 2^2\cdot 2^{-2x}-\frac{1}{2}\cdot 9\cdot 3^{-2x}$$

$$3\cdot 2^{-2x}+27\cdot 3^{-2x}=24\cdot 2^{-2x}-\frac{9}{2}\cdot 3^{-2x}$$

Substituce: $-2x=y$

$$3\cdot 2^y+27\cdot 3^y=24\cdot 2^y-\frac{9}{2}\cdot 3^y$$

$$27\cdot 3^y+\frac{9}{2}\cdot 3^y=24\cdot 2^y-3\cdot 2^y$$

$$3^y\left(27+\frac{9}{2}\right)=2^y\cdot(24-3)$$

$$\frac{63}{2}\cdot 3^y=21\cdot 2^y$$

$$\frac{3^y}{2^y}=\frac{\frac{21}{1}}{\frac{63}{2}}$$

$$\left(\frac{3}{2}\right)^y=\frac{2}{3}$$

$$\left(\frac{3}{2}\right)^y=\left(\frac{3}{2}\right)^{-1}$$

$$y=-1$$

Zpět do sub.

$$-2x=-1$$

$$\boxed{x=\frac{1}{2}}$$

Zkouška:

$$L=3\cdot 4^{-\frac{1}{2}}+\frac{1}{3}\cdot 9^{\frac{3}{2}}=3\cdot\frac{1}{\sqrt{4}}+\frac{1}{3}\sqrt{9^3}=$$

$$3\cdot\frac{1}{2}+\frac{1}{3}\cdot 27=\frac{3}{2}+9=\frac{21}{2}$$

$$P=6\cdot 4^{\frac{1}{2}}-\frac{1}{2}\cdot 9^{\frac{1}{2}}=6\cdot\sqrt{4}-\frac{1}{2}\sqrt{9}=$$

$$6\cdot 2-\frac{1}{2}\cdot 3=12-\frac{3}{2}=\frac{21}{2}$$

$$L=P$$

Řešte rovnici:

$$\left(\frac{1}{5}\right)^{x-1}-\left(\frac{1}{5}\right)^{x+1}=\frac{24}{5}$$

$$\left(\frac{1}{5}\right)^x\cdot\left(\frac{1}{5}\right)^{-1}-\left(\frac{1}{5}\right)^x\cdot\frac{1}{5}=\frac{24}{5}$$

$$\left(\frac{1}{5}\right)^x\cdot 5-\left(\frac{1}{5}\right)^x\cdot\frac{1}{5}=\frac{24}{5}$$

$$\left(\frac{1}{5}\right)^x\cdot\left(5-\frac{1}{5}\right)=\frac{24}{5}$$

$$\left(\frac{1}{5}\right)^x\cdot\frac{24}{5}=\frac{24}{5}$$

$$\left(\frac{1}{5}\right)^x=\frac{24}{5}\cdot\frac{5}{24}$$

$$\left(\frac{1}{5}\right)^x=1$$

$$\left(\frac{1}{5}\right)^x=\left(\frac{1}{5}\right)^0$$

$$x=0$$

Zkouška:

$$L=\left(\frac{1}{5}\right)^{-1}-\left(\frac{1}{5}\right)^1=5-\frac{1}{5}=\frac{24}{5}$$

$$P=\frac{24}{5};\ L=P$$

Řešte rovnici:

$$9^{\sqrt{x+2}}=27\cdot 3^{\sqrt{x+2}}$$

Sub. $\sqrt{x+2}=y$

$$9^y=27\cdot 3^y$$

$$\frac{9^y}{3^y}=27$$

$$3^y=27$$

$$3^y=3^3$$

$$y=3$$

Zpět:

$$\sqrt{x+2}=3$$

$$x+2=9$$

$$\boxed{x=7}$$

Zkouška:

$$L=9^3=729$$

$$P=27\cdot 3^3=729;\ L=P$$

Řešte pomocí: ⊛⊛

$$\frac{1}{3^4}\cdot(3^x)^{x+2}=(\sqrt{27})^x\cdot\frac{\sqrt{27^x}}{9}$$

$$3^{-4}\cdot3^{x^2+2x}=(\sqrt{3^3})^x\cdot\frac{\sqrt{(3^3)^x}}{3^2}$$

$$3^{-4}\cdot3^{x^2+2x}=(3^{\frac{3}{2}})^x\cdot\frac{\sqrt{3^{3x}}}{3^2}$$

$$3^{(x^2+2x-4)}=3^{\frac{3x}{2}}\cdot3^{\frac{3x}{2}}\cdot3^{-2}$$

$$3^{(x^2+2x-4)}=3^{3x-2}$$

$$x^2+2x-4=3x-2$$

$$x^2-x-2=0$$

$$x_{1,2}=\frac{1\pm\sqrt{1+8}}{2}=\frac{1\pm3}{2}= \boxed{x_1=2},\ \boxed{x_2=-1}$$

Zkouška pro x_1:

$L=\frac{1}{3^4}\cdot(3^2)^4=3^{-4}\cdot3^8=3^4=81$; $P=(\sqrt{27})^2\cdot\frac{\sqrt{27^2}}{9}=27\cdot\frac{27}{9}=27\cdot3=81$; $L=P$

Zkouška pro x_2:

$L=3^{-4}\cdot(3^{-1})^1=3^{-4}\cdot3^{-1}=3^{-5}$

$P=(\sqrt{27})^{-1}\cdot\frac{\sqrt{27^{-1}}}{9}=\frac{1}{\sqrt{27}}\cdot\frac{\sqrt{\frac{1}{27}}}{9}=$

$=\frac{1}{\sqrt{27}}\cdot\frac{\frac{1}{\sqrt{27}}}{\frac{9}{1}}=\frac{1}{\sqrt{27}}\cdot\frac{1}{9\cdot\sqrt{27}}=\frac{1}{9\cdot27}=\frac{1}{3^2\cdot3^3}=$

$=\frac{1}{3^5}=3^{-5}$; $L=P$

⊛⊛ Řešte v R:

$$\sqrt{3^x}\cdot(3^{x-1})^{x+1}=\frac{1}{\sqrt[4]{9^{x-2}}}$$

$$3^{\frac{x}{2}}\cdot3^{x^2-1}=\frac{1}{\sqrt[4]{(3^2)^{x-2}}}$$

$$3^{(x^2+\frac{1}{2}x-1)}=\frac{1}{\sqrt[4]{3^{2x-4}}}$$

$$3^{(x^2+\frac{1}{2}x-1)}=\frac{1}{3^{\frac{2x-4}{4}}}$$

$$3^{(x^2+\frac{1}{2}x-1)}=3^{(\frac{1}{2}x-1)}$$

$$3^{(x^2+\frac{1}{2}x-1)}=3^{-\frac{1}{2}x+1}$$

$$x^2+\frac{1}{2}x-1=-\frac{1}{2}x+1$$

$$x^2+x-2=0$$

$$x_{1,2}=\frac{-1\pm\sqrt{9}}{2}=\frac{-1\pm3}{2}= \boxed{x_1=1},\ \boxed{x_2=-2}$$

Zkouška pro x_1:

$L=\sqrt{3}\cdot(3^0)^2=\sqrt{3}\cdot3^0=\sqrt{3}\cdot1=\sqrt{3}$

$P=\frac{1}{\sqrt[4]{9^{-1}}}=\frac{1}{9^{-\frac{1}{4}}}=\frac{1}{(3^2)^{-\frac{1}{4}}}=\frac{1}{3^{-\frac{1}{2}}}=$

$=3^{\frac{1}{2}}=\sqrt{3}$; $L=P$

Zkouška pro x_2

$L=\sqrt{3^{-2}}\cdot(3^{-3})^{-1}=3^{-\frac{2}{2}}\cdot3^3=3^{-1}\cdot3^3=3^2$

$=9$; $P=\frac{1}{\sqrt[4]{9^{-4}}}=\frac{1}{9^{-\frac{4}{4}}}=\frac{1}{9^{-1}}=$

$=9$; $L=P$

Řešte v R: $5^x\cdot4^{1-x}-4^x\cdot5^{1-x}=1$

4. $5^x\cdot4\cdot4^{-x}-4^x\cdot5\cdot5^{-x}=1$, sh. 17

$$4\cdot 5^x\cdot\frac{1}{4^x}-5\cdot 4^x\cdot\frac{1}{5^x}=1$$

$$4\cdot\frac{5^x}{4^x}-5\cdot\frac{4^x}{5^x}=1$$

$$4\cdot\left(\frac{5}{4}\right)^x-5\cdot\left(\frac{5}{4}\right)^{-x}=1$$

$$4\cdot\left(\frac{5}{4}\right)^x-5\cdot\left[\left(\frac{5}{4}\right)^x\right]^{-1}=1$$

Substituce: $\left(\frac{5}{4}\right)^x=y$

$$4y-5\cdot(y)^{-1}=1$$

$$4y-\frac{5}{y}-1=0\quad|\cdot y$$

$$4y^2-5-y=$$

$$4y^2-y-5=0$$

$$y_{1,2}=\frac{1\pm\sqrt{1+81}}{8}=\frac{1\pm\sqrt{81}}{8}=\frac{1\pm 9}{8}=\left\langle\begin{array}{l}y_1=\frac{5}{4}\\ y_2=-1\end{array}\right.$$

Zpět do sub.

$\left(\frac{5}{4}\right)^x=\frac{5}{4}$ $\qquad$ $\left(\frac{5}{4}\right)^x=-1$ nemá řešení

$$\left(\frac{5}{4}\right)^x=\left(\frac{5}{4}\right)^1$$

$$\boxed{x=1}$$

Zkouška:

$L=5^1\cdot 4^0-4^1\cdot 5^0=5\cdot 1-4\cdot 1=5-4=1$

$P=1\ \dots\ L=P$

⊗⊗ Řešte rovnici:

$$\left(\frac{5}{8}\right)^{\frac{2x+1}{x-1}}=\left(\frac{512}{125}\right)^{3-x}$$

$$\left(\frac{5}{8}\right)^{\frac{2x+1}{x-1}}=\left(\frac{8^3}{5^3}\right)^{3-x}$$

$$\left(\frac{5}{8}\right)^{\frac{2x+1}{x-1}}=\left[\left(\frac{8}{5}\right)^3\right]^{3-x}$$

$$\left(\frac{5}{8}\right)^{\frac{2x+1}{x-1}}=\left[\left(\frac{5}{8}\right)^{-3}\right]^{3-x}$$

$$\left(\frac{5}{8}\right)^{\frac{2x+1}{x-1}}=\left(\frac{5}{8}\right)^{-9+3x}$$

$$\frac{2x+1}{x-1}=-9+3x$$

$$2x+1=(x-1)\cdot(-9+3x)$$

$$2x+1=-9x+9+3x^2-3x$$

$$3x^2-14x+8=0$$

$$x_{1,2}=\frac{14\pm\sqrt{100}}{6}=\frac{14\pm 10}{6}=\left\langle\begin{array}{l}\boxed{x_1=4}\\ \boxed{x_2=\frac{2}{3}}\end{array}\right.$$

Zkouška pro x_1:

$$L=\left(\frac{5}{8}\right)^{\frac{9}{3}}=\left(\frac{5}{8}\right)^3$$

$$P=\left(\frac{512}{125}\right)^{-1}=\frac{125}{512}=\frac{5^3}{8^3}=\left(\frac{5}{8}\right)^3;\ L=P$$

Zkouška pro x_2:

$$L=\left(\frac{5}{8}\right)^{\frac{\frac{7}{3}}{-\frac{1}{3}}}=\left(\frac{5}{8}\right)^{-7}=\left(\frac{8}{5}\right)^7;\ P=\left(\frac{512}{125}\right)^{\frac{7}{3}}=\left(\frac{8^3}{5^3}\right)^{\frac{7}{3}}=\frac{8^7}{5^7}=\left(\frac{8}{5}\right)^7$$

$L=P$