Universal Journal manuscript no. qlm˙univerzal
March 20, 2007

Quark-Lepton Model based on the Generalized Planck Scales and the Fibonacci sequence

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Received March 1, 2007; accepted March 16, 2007

ABSTRACT

Aims. We present an empirical formulae that are based on the generalized Planck scales and the Fibonacci numbers. These relationships allow to calculate the masses of elementary particles and therefore we called our model the quark-lepton model (QLM).
Methods. We tested variety of functions such as Legendre or Jacobi polynomials, Bessel functions, Hermit and Cebysev polynomials, Finonacci sequences and other to obtain the function for the quantum coefficient κ that together with the generalized Planc scales provides the masses of elementary particles.
Results. We have discovered the empirical formulae that appear from Fibonacci sequences and involving quantum numbers $(0, \pm1, \pm2, \pm3)$. Our QLM generates the value of elementary particles which are with perfect harmony with measured ones and predict two as many as four new particles.

Key words. generalized Planck scales – quark-lepton model – Fibonacci sequence

1. Introduction

In the recent work (Kodejska 2007) we outlined the possibility to calculate an arbitrary mass of a body or a particle. This power law (called generalized Planck scales) based on the fundamental constants $\{h, c, G, \alpha\}$ allow to describe macroskopic (the universe) as well as microspopic objects (elementary particles) by the aid of the coefficient from a very close interval $\langle -33; 33 \rangle$.

Recently Breakstone (2006) derived empirical relationships among elementary fermion masses based on simple exponential formulae involving quantum numbers for the electromagnetic and strong interactions, with a weak correction factor motivated by a simple linear combination of mass terms for the weak and electromagnetic interactions of charged leptons. He has found also a relationship among neutrino mass ratios and extended its to the quark sector. Nevertheless, these results where obtained on the assumption that the electron mass is measured and not calculated.

In the following we show that generalized Planck scales and the Fibonacci sequence form the basis for quark-lepton model in which the massess of the all particles are calculated without any measured value of the mass.

2. Generalized Planck Scales - GPS

As we derived in (Kodejska 2007) , for a mass-scale we can get

$$m(\kappa) = h^{\frac{1}{2}-\kappa} c^{\frac{1}{2}+\kappa} G^{-\frac{1}{2}} e^{2\kappa} \mu_0^{\kappa}, \tag{1}$$

or in the best form:

$$m(\kappa) = \sqrt{\frac{hc}{G}} \left(\frac{\mu_0 c e^2}{h} \right)^{\kappa} = m_{\mathrm{P}} \cdot (2\alpha)^{\kappa}. \tag{2}$$

If we take a logarithmic form than Eq.(2) yields

$$\ln \left(\frac{m(\kappa)}{m_{\mathrm{P}}} \right) = \kappa \cdot \ln (2\alpha), \tag{3}$$

where m_{P} is Planck mass, α is the fine structure constant and κ is so-called *the quantum coefficient*.

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3. Physical Meaning of GPS

As we can see from the equations (1),(2) or (3), these formulae can be interpreted as a power law for arbitrary mass (from elementary particles to the whole universe) which is only created from the fundamental constants of nature such as $\{h, c, G, \alpha\}$ and some real number κ called *the quantum coefficient*.

The relatively close range for a value of κ can be estimated as an interval $\langle -33; 33\rangle$ where (-33) corresponds to the whole universe and right side of an interval, (33) is a lower limit for masses of elementary particles, for example.

Some significant values of the quantum coefficient κ are presented in the Table 1.

Table 1. Quantum coefficient κ in relation to the rough masses of objects

κ	mass (kg)	object
-33	10^{53}	universe
-25	10^{40}	massive black holes
-20	10^{30}	Sun
-20 to -15	10^{25}	planets of solar system
-5	10^{2}	people
10 to 15	10^{-30}	elementary particles

We can see that the main significance of GPS is in a coupling between the world of elementary particles and macroscopic objects including our universe through a very close interval for the quantum coefficient.

3.1. Application of GPS to elementary particles

If we take into account the charged leptons and apply to their masses Eq.(3) than we can get for the quantum coefficient following results which are given in Table 2.

Table 2. Quantum coefficient κ in relation to the charged leptons

particle	mass (MeV)	κ
e	0.510998918(44)	12.40730164
μ	105.6583692(94)	11.14601064
τ	1776.99(29)	10.47830301

If we find some function that generates the values for the quantum coefficient κ such as mentioned in Table 2 we can use Eq.(2) to get the masses of all elementary particles. Note, that the value of κ is approximately in the interval $\langle 9; 17\rangle$.

3.2. κ- function requirements

What properties of κ-function should have been takeing into account? We suppose that the basic characteristics are as follows:

- κ-function must be sufficient in numbers of elementary particles, thus say for example about 12 particles,
- κ-function must generate such values that are depending on the quantum numbers $(0, \pm1, \pm2, \pm3, \ldots)$,
- κ-function must have upper and lower limit that cutting-down number of elementary particles,
- κ-function must provide the values for the masses of the elemntary particles according to measured ones.

We tested a large number of function such as Legendre or Jacobi polynomials, Bessel functions, Hermit and Cebysev polynomials, various sequences to obtain the best results for the masses of the elementart particles. Note, that we found certain κ-function based on the Fibonacci sequence which may not be the right one but which provides the values in a good agreement with observable ones. Probably, κ-function has an other expression and our result described in the following section is only the curve fitting for the regular function.

4. Quark-Lepton Model

In the following we use the values of the fundamental constants as $h = 6.6260693 \times 10^{-34}$ kg m^2 s^{-1}, $c = 299792458$ m s^{-1}, $G = 6.6742 \times 10^{-11}$ kg^{-1} m^3 s^{-2}, $\alpha = 7.297352568 \times 10^{-3}$, $m_{\mathrm{u}} = 1.66053886 \times 10^{-27}$ kg $= 931.4940495$ MeV/c^2, $\varphi = 1.6180339$, where h is Planck constant, c is speed of light, G represents Newtonian constant of gravitation, α is fine structure constant, m_{u} is atomic mass unit and φ is the limit of the Fibonacci sequence. The values of these constants (except φ) have been adopted from (Mohr & Taylor 2005).

We derived for the quantum coefficient κ following empirical formulae:

$$\kappa = \left[\frac{F_{12+n+1} + sgn[2n-3]\cdot F_{8+sgn[n]\cdot 2^{|n|}-1\cdot sgn[n]} + \varphi^{-(n+1)} - A}{F_{12+n+2} + F_{12+n} + B + cos^2(n\cdot\frac{\pi}{2})}\right]^{sgn[1-2n]} + (12+n)\,, \tag{4}$$

where

$$F_k = \frac{1}{\sqrt{5}} \cdot \left[\left(\frac{1+\sqrt{5}}{2} \right)^k - \left(\frac{1-\sqrt{5}}{2} \right)^k \right], \tag{5}$$

$$A = \frac{\varphi^{-(n+1)}}{(1-2n) \cdot F_{3n \cdot sgn[n] + (1-n) \cdot F_{5-n \cdot sgn[n]}} + F_{12+n-1}}, \tag{6}$$

$$B = \{b_1; b_2; b_3; b_4\}, \tag{7}$$

whereas

$$b_2 = 2 \cdot F_{17+4n} - F_{19+4n}, \tag{8}$$

$$b_1 = b_2 - 24, \tag{9}$$

$$b_3 = F_{18+3n} - 2 \cdot F_{8+n} + (n-1), \tag{10}$$

$$b_4 = -b_3 - [(12+4n) \cdot b_2 + (3+n) \cdot b_1], \tag{11}$$

$$\tag{12}$$

where F_k are Fibonacci numbers and A, B are some correcting functions. The numerical results of this calculation are given in Table 3.

Table 3. Quark-Lepton Model

n	model	A	B	κ	calc. mass (MeV)	particle	measured mass (MeV)
-3	$\frac{F_{10}-F_1+\varphi^2-A}{F_{11}+F_9+B+0} + 9$	$\frac{\varphi^2}{7 \cdot F_{13}+F_8} = \frac{\varphi^2}{1652}$	$b_3 : F_9 - 2 \cdot F_5 - 4 = 20$	9.395919225	172 476.4875	t-quark	172 000 ± 1700 ± 2400
			$b_2 : 2 \cdot F_5 - F_7 = -3$	9.471803743	125 147.0872	NEW	—
			$b_1 : b_2 - 24 = -27$	9.589754679	76 012.6488	$W^{\pm}$	? boson ?
			$b_4 : -b_3 = -20$	9.549674264	90 046.2601	Z^0	? boson ?
-2	$\frac{F_{11}-F_5+\varphi^1-A}{F_{12}+F_{10}+B+1} + 10$	$\frac{\varphi^1}{5 \cdot F_{12}+F_9} = \frac{\varphi^1}{754}$	$b_3 : F_{12} - 2 \cdot F_6 - 3 = 125$	10.263433501	4 407.0009	b-quark	4 260 ± 150 · 2
			$b_2 : 2 \cdot F_9 - F_{11} = -21$	10.478301051	1 777.0035	τ	1 776.99(29)
			$b_1 : b_2 - 24 = -45$	10.552360568	1 299.3600	c-quark	1000 – 1400
			$b_4 : \ldots = 4$	10.419685725	2 276.6411	NEW	heavy c-quark
-1	$\frac{F_{12}-F_7+\varphi^0-A}{F_{13}+F_{11}+B+0} + 11$	$\frac{\varphi^0}{3 \cdot F_9+F_{10}} = \frac{1}{157}$	$b_3 : F_{15} - 2 \cdot F_7 - 2 = 582$	11.146010653	105.6583618	μ	105.6583692(94)
			$b_2 : 2 \cdot F_{13} - F_{15} = -144$	11.741537250	8.5238022	d-quark	5 – 9
			$b_1 : b_2 - 24 = -168$	11.857101497	5.2297387	u-quark	1.5 – 5
			$b_4 : \ldots = 906$	11.107486669	124.3444432	s-quark	80 – 155
0	$\frac{F_{13}-F_8+\varphi^{-1}-A}{F_{14}+F_{12}+B+1} + 12$	$\frac{\varphi^{-1}}{1 \cdot F_5+F_{11}} = \frac{\varphi^{-1}}{94}$	0	12.407301645	0.510998907	e	0.510998918(44)
1	$\left[\frac{F_{14}-F_9+\varphi^{-2}-A}{F_{15}+F_{13}+0+0}\right]^{-1} + 13$	$\frac{\varphi^{-2}}{-1 \cdot F_3+F_{12}} = \frac{\varphi^{-2}}{142}$	0	15.455011291	1.298435427 eV	ν_τ	???
2	$\left[\frac{F_{15}+F_{11}+\varphi^{-3}-A}{F_{16}+F_{14}+0+1}\right]^{-1} + 14$	$\frac{\varphi^{-3}}{-3 \cdot F_4+F_{13}} = \frac{\varphi^{-3}}{224}$	0	15.952133363	0.158782024 eV	ν_μ	???
3	$\left[\frac{F_{16}+F_{15}+\varphi^{-4}-A}{F_{17}+F_{15}+0+0}\right]^{-1} + 15$	$\frac{\varphi^{-4}}{-5 \cdot F_7+F_{14}} = \frac{\varphi^{-4}}{312}$	0	16.381840349	0.025819228 eV	ν_e	< 0.06 eV

The measured masses come from CODATA 2002 and PDG 2004, (Mohr & Taylor 2005; Eidelman et al. 2004), the value for the electron neutrino is from (Hannestad 2002b).

4.1. Discussion of obtained results

The results obtained by κ-function are in very good agreement with the measured masses of elementary particles. Nevertheless, neutrino mass bounds arise from the cosmological considerations as mentioned by Hannestad (2002a;2002b;2004). The upper limit for a total mass of all neutrinos is determined as $\sum m_\nu < 1.8$ eV. In our Q-L model the total neutrino mass is calculated as $\sum m_\nu = 1.48$ eV.

The quark masses realized from our Q-L model correspond to the measured ones as well as to other theoretical works (Koide 1994; Breakstone 2006). For example, Breakstone (2006) calculated the mass of the s-quark to be 124.0(2.1) MeV and our result is 124.34 MeV, Koide (1994) mentioned the values for u-quark and d-quark as 5.6 ± 1.1 MeV and 9.9 ± 1.1 MeV, respectively. We calculated for u-quark 5.2 MeV and for d-quark 8.5 MeV.

5. Conclusion

As noted throughout this paper, foregoing calculations are empirically derived. There is, unfortunately, no theoretical understanding of the form of the equations. To explain why the main index in κ-function is equal to 12 there have been string theory motivated results take into account (number of dimensions of a brane, for instance). As with generalized Planck scales, this work makes a definite and accurate prediction of the e, μ and τ masses.

For the neutrinos, future oscillation experiments or cosmological observations will provide accurate values for m_e, m_μ and m_{tau}.

For the quarks, these calculations give qualitative agreement with measured values of the known quarks and predict two new quarks operatively entitled as *heavy c-quark* with the mass 2 276 MeV and *light t-quark* with the mass 125 147 MeV. There is clearly additional work needed to understand whether or not this approch has validity.

To summarize, the theory of the quantum coefficient needs to be developed to obtain the best function that describes not only quarks and leptons but all particles including baryons, mesons and bosons. Hopefully this work is the first step towards a deeper understanding of the values of fermion and boson masses.

References

Breakstone, A. 2006, Empirical Relationships among Lepton and Quark Masses, physics/0602118
Eidelmann, S. et al. 2004, Particle Data Group, Phys. Lett. B 592,1
Goobar, A., Hannestad, S., Mortsell, E., Tu, H. 2006, The neutrino mass bound from WMAP-3, astro-ph/0602155
Hannestad, S., 2002a, Cosmological limit on the neutrino mass, astro-ph/0205223
Hannestad, S., 2002b, Can cosmology detect hierarchical neutrino masses?, astro-ph/0211106
Hannestad, S., 2004, Neutrino mass bounds from cosmology, astro-ph/0412181
Koide, Y., 1994, Table of Running Quark Mass Values:1994, astro-ph/9410270
Kodejska, C., 2007, Generalized Planck Scales and Possible Cosmological Consequneces, http://delta.ssakhk.cz/matfyz/dokumenty/papers/gpsapcc.pdf
Mohr, P. J., & Taylor, B. N. 2005, CODATA - values of the fundamental constants, Rev. Mod. Phys., Vol. 77, No.1